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THE GROWTH OF THE HIGH-FREQUENCY ELECTRODELESS DISCHARGE

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CONTENTS

In this paper we consider only discharges in gases at such low pressure that the mean free path of electrons is greater than the size of the vessel.

Part I. The elementary theory of the starting of an electrodeless discharge in a gas at low pressure by a uniform high-frequency electric field has been given by Gill & von Engel (1948). It was shown that a discharge will start when the applied field is large enough to cause multiplication of electrons by secondary electron emission from the end-walls of the vessel; initially, gas ionization is absent. Multiplication occurs if an electron starting at one end-wall with a small energy and in a suitable phase of the applied field crosses the vessel in one half-cycle and hits the opposite wall fast enough to release more than one secondary electron. Secondary electrons usually start in a negative phase

of the field, but escape from the wall because of their initial energy, unless the phase is more negative than a certain limiting value; this corresponds to the cut-off wave-length, i.e. the longest one at which a discharge can be started.

Here, for the first time, the growth of the discharge is treated in detail, the calculations being based on known atomic data only. When secondary electrons leave an end-wall a positive wall charge is left behind, which retards the electrons. This is important only near the cut-off wave-length. These wall charges cause the phase at one wall to become increasingly negative until, finally, the electrons would fail to escape, and the multiplication would cease, which is contrary to experience. However, the growth can be explained by considering the velocity distribution of the secondary electrons. Then a distribution in phase ensues, which must be repeated in successive half-cycles for an avalanche to develop. During this first stage the current is therefore essentially controlled by secondary emission, and grows exponentially with time.

At these low pressures electrons rarely collide with gas molecules. Thus the electrons must make many transits across the vessel to form a large number of positive ions. The ions remain almost stationary in the gas; they are nearly uniformly distributed although slightly concentrated at the centre of the vessel. A second stage in the growth of the discharge begins when the ion space charge first appreciably affects the motion of the electrons. Although electrons are still produced mainly at the end-walls, the rate steadily decreases as the ion space charge grows. The rate of production of ions and electrons in the gas also decreases, and losses of both ions and electrons due to self-repulsion become important. The current thus rises more slowly than it would if space charges did not develop, until it reaches a constant value.

It is shown that at very low pressures this second stage may not be reached, because self-repulsion of the electrons stops the development earlier. The final equilibrium state for larger pressures is not included in this treatment.

This theory predicts the dependence of the growth on the material of the walls, on the nature of the gas and its pressure, and the effect of a field greater than the starting field.

Part II. A new experimental technique has been employed to measure the current actually flowing through the discharge. The large capacitative current flowing across the external electrodes is balanced out by a bridge method, the bridge becoming unbalanced when a current flows through the gas. The unbalanced component is proportional to the discharge current and is amplified, rectified and displayed on an oscilloscope. In order to measure the growth of this current with time, pulses of high-frequency potential are applied across the discharge vessel, and the time-base of the oscilloscope synchronized with the pulses.

Oscillograms show how the growth of current depends upon the pressure (2 to 35μ) and nature of the gas (hydrogen and helium), the excess voltage applied ($\leqslant 20\,\%$) and the frequency of the applied field (10 to 20 Mc/s). The spatial distribution of light from the discharge in the final state is also measured, from which the motion of the electrons can be deduced, and compared with theory.

Good agreement is obtained between theoretical predictions derived solely from atomic data, and experimental results.

These investigations clearly demonstrate that at low pressure the properties of the wall mainly control the multiplication process in the initial stage and thus the starting field, whereas the properties of the gas become important in the later stages and so essentially determine the total time of growth of the discharge.

PART I. THEORY

1. Introduction

It has been known for almost half a century that a gas can be made conducting when placed in a rapidly alternating electric field. Some of the earliest experiments were made by Tesla. A systematic study of high-frequency gas discharges did not begin until about 1920 (see Loeb 1939). The first high-frequency discharge to be extensively examined was the electrodeless ring discharge which J. J. Thomson (1933) thought started when the electrons circling in a high-frequency magnetic field acquired ionization energy. In this paper we are concerned only with discharges produced in an electric field. Most of the early experiments were done at low pressures (up to a few centimetres of mercury), and at frequencies up to a few megacycles per second. The variation was observed of the starting or maintenance potential with the nature and pressure of the gas, the frequency of the applied field, and the electrode separation. Some workers used internal electrodes (Kirchner 1930), while others (Gutton 1928) placed electrodes outside the discharge tube, thus exciting the so-called electrodeless discharge. These external electrodes were usually movable cylindrical sleeves fitted over the ends of cylindrical tubes. Not much thought was given to the resulting non-uniformity of the electric field in the gas, and its consequences on any theoretical interpretation of the results.

Several ideas were advanced to explain the mechanism of such a discharge. At first it was thought to be analogous to a d.c. glow discharge; each electrode or the nearby end-walls acts as a cathode in alternate half-cycles of the applied field, and produces the negative glow and the cathode fall, while the positive column fills the centre of the discharge tube. Later theories (e.g. Townsend & Gill 1938) considered only the motion of free electrons in the gas under the influence of an alternating electric field ignoring all wall and electrode processes and space-charge effects. J. Thomson (1937) derived an elementary theory of this type, regarding each electron as oscillating about a mean position in the gas. In order to ionize the gas each electron must, at some point in its path, attain enough energy and, moreover, it must hit a molecule before it returns this energy again to the field. Only electrons which start with zero initial velocity, and in zero phase of the field (or with certain combinations of velocity and phase), can perform this motion; and these can only be a minute fraction of all the electrons present. Another difficulty is that when an electron collides with a molecule elastically or inelastically, it suffers a deflexion or a loss of energy interrupting the regular oscillation; this effect also is not discussed in the theory. For the same reason the amplitude of oscillation of an electron must be small compared with the dimensions of the vessel. These conditions effectively limit the field strength, the frequency, the size of the vessel and the pressure of the gas. Moreover, such a treatment cannot describe the mechanism of discharges at very low pressure when the mean free path of electrons is comparable with the dimensions of the vessel. Gill & Donaldson (1931) recognized that the motion of an electron in an alternating field depends on the phase in which the electron starts, the motion in general being an oscillation superimposed on an accelerated motion in one direction.

From observed values of the starting field (about 20 V/cm) Brasefield (1931) concluded that electrons cannot ionize the gas. Hiedemann (1931), however, pointed out that an electron can attain ionization energy if it is deflected in an elastic collision and subsequently accelerated by the field.

Several workers observed a double minimum in the graph of the starting potential as a function of the pressure. This occurred only at lower frequencies and disappeared when the frequency was increased. Gill & Donaldson showed that this was connected with the dimensions and shape of the vessel. At low frequencies the electrons oscillate with large amplitude and losses to the walls may be large. It became clear that no theory could be adequate which considered only ionization in the gas caused by oscillating electrons hitting gas molecules, while ignoring all wall and electrode processes, and the presence of the positive-ion space charge. Moreover, a reasonable comparison between theory and experiment could only be expected if uniform electric fields were used, which could be accurately measured.

A new approach to the solution of these problems was made in the experiments of Gill & von Engel (1948), who measured the starting potential of a high-frequency discharge as a function of the wave-length of the applied field, in gases at very low pressure, that is, of order 10^{-3} mm Hg. Cylindrical glass vessels were placed between two parallel plates in a uniform electric field directed along the axis of the vessel. The mean free path of electrons at these pressures is much greater than the length of the vessels used. It was found that the starting field is independent of the gas and the pressure, but depends upon the wave-length of the applied field, and the dimensions and material of the vessel, although the fully developed discharge shows the spectrum of the gas. It was also observed that for a vessel of given length d, there is a maximum (cut-off) wave-length λ_c beyond which no field, however large, can start a discharge.

From these results Gill & von Engel deduced a theory of the initiation of the discharge. They showed that an initial electron produced in the vessel by some chance process can be multiplied by secondary emission from the end (glass) walls. An electron which starts, say, near one end-wall must traverse the tube in half a cycle, and hit the opposite end-wall with a speed which is large enough to release more than one secondary electron. The secondary electrons then repeat this motion exactly in the opposite direction. These two conditions—namely, that the transit time is equal to half a cycle and the final energy of impact greater than the critical value mentioned above—determine not only the field but also the phase in which an electron starts. As the wave-length increases, the phase in which an electron must start at one end-wall becomes steadily more negative, until a critical value of λ is reached when the secondary electrons, in spite of their small initial velocity, are driven back on to the wall and lost. Thus no multiplication or discharge can occur when $\lambda \geqslant \lambda_c$. Although no attempt has been made to describe the development of gas ionization, the theory shows that the maximum energy which an electron can attain in the field X is proportional to $X^2\lambda^2$, and that in the frequency range studied, this energy is much greater than the ionization energy of any gas, though of the same order. Therefore ionization can occur by collisions between electrons and molecules. The development of a discharge of this type is now considered in further detail.

Numerical results of the theory throughout this paper are calculated for discharges in hydrogen and helium at certain pressures in a cylindrical glass vessel of specified size; these will be compared in part II of this paper with experimental results obtained in these conditions.

2. Multiplication by secondary emission

On the simple theory advanced by Gill & von Engel the number of electrons is multiplied by a factor δ' every time they hit an end-wall. This factor δ' is the secondary emission coefficient, defined as the number of secondary electrons which are released by one primary electron of known energy. Its value has been measured for various kinds of glass (Salow 1940; Mueller 1945). δ' must exceed 1 for the number of electrons to increase. If we assume that the secondary electrons start from one end-wall with an initial energy of a few electron volts in a phase ϕ of the applied field $X \sin(\omega t + \phi)$, then a simple calculation, using known values of X and ϕ, shows that they hit the opposite wall with an energy of about 90 eV. The corresponding value of δ' for Pyrex glass is about 1·4. But since the value of the field X used in this calculation is the least which will start the discharge one would expect the multiplication factor to be only very slightly greater than 1. If it were not so the discharge would start with a lower applied field; and a slight change of the phase in which the electrons start would maintain the transit time at exactly one half-cycle. This apparent discrepancy may be accounted for by losses of electrons to the walls.

We must therefore distinguish between an effective multiplication factor δ which occurs in the particular conditions of the discharge, and the true secondary electron coefficient δ' as measured by Salow and Mueller. The difference between δ and δ' arises from the angular distribution of secondary electrons resulting in losses to the side walls and other causes which will be discussed later on. Thus δ' is always larger than δ. When X is the starting field, δ just exceeds 1.

Assume that the discharge is started by one chance electron created, e.g. by cosmic rays or radioactive surroundings in favourable conditions, that is, near one end-wall, and in the correct phase of the field. The number of electrons present after n half-cycles, i.e. after n impacts with the end-walls, will be δ^n. Since n is proportional to the time t which has elapsed since the chance electron was created, the number of electrons and hence the current through the discharge tube will be proportional to $\delta^{\alpha t}$.

This process cannot continue indefinitely, for the current would tend rapidly to infinity. Some other effect must become dominant to reduce the rate of growth of current, and finally to establish an equilibrium state. There are several possible causes for the current departing from a $\delta^{\alpha t}$ law:

(*a*) When the number of electrons becomes very large many of them may be driven to the walls by their own field. This self-repulsion might lead to an equilibrium state when $\delta = 1$.

(*b*) Even at low pressure there will be some ionizing collisions between electrons and gas molecules, and the number of ions formed will increase as the number of electrons increases. Since the ions are heavy they will move very little under the influence of the applied field. A positive space charge will therefore develop in the vessel, thus distorting the applied field and reducing the rate of multiplication.

(*c*) Whenever electrons are multiplied by secondary emission at an end-wall, a positive charge is left there, since more electrons leave the wall than hit it. This positive charge will retard the secondary electrons leaving the wall; the transit time will become more than half a cycle, and after the next impact the secondary electrons will leave the end-wall in a more positive phase of the field. The combination of this change in phase of the electrons, and the retarding effect of the wall charges, might affect the rate of growth of current.

The positive wall charges will affect the motion of the electrons very early in the growth of the discharge as will be shown later. At the same time, positive ions will be produced in the gas, and their space charge will at a later stage alter the rise of current. Self-repulsion of electrons will become important when their concentration is large.

3. The first stage of the discharge

(a) *Growth of the number of electrons and ions*

Assume that the electrons have multiplied by secondary emission to some relatively small number m (say 50) after which statistical fluctuations, whether in the multiplication factor δ or due to collisions between electrons and molecules, may be neglected. Suppose these m electrons all leave an end-wall at one instant in a phase ϕ of the field, each with the same initial velocity v_0. A small fraction a of them will hit and ionize gas molecules in their transit across the vessel; these electrons are assumed to be lost to the walls because they fall out of phase with the main body. Thus only a number $m(1-a)$ of electrons reach the opposite wall, and are multiplied by secondary emission to $\delta m(1-a)$. By this process the wall acquires a positive charge which is equal to the difference between the negative charges leaving and those arriving at the wall, that is,

$$\delta m(1-a)-m(1-a) = m(\delta-1)(1-a).$$

This charge is added to the positive charge already accumulated on this wall from previous impacts. Neglecting statistical variations it is convenient to calculate the increase in the number of electrons and ions starting from the first chance electron; the results obtained will be most true when the number of charges becomes large. The calculation is shown in table 1. σ_1, σ_2 are the surface densities of charge on the two walls.

Table 1. Multiplication of electrons, ions and wall charges

no. of half-cycles, n	no. of electrons leaving the wall	no. of new ions formed during the half-cycle	contribution to σ_1	contribution to σ_2
1	1	a	0	0
2	$\delta(1-a)$	$a\delta(1-a)$	0	$(\delta-1)(1-a)e$
3	$\delta^2(1-a)^2$	$a\delta^2(1-a)^2$	$(\delta-1)\delta(1-a)^2 e$	0
4	$\delta^3(1-a)^3$	$a\delta^3(1-a)^3$	0	$(\delta-1)\delta^2(1-a)^3 e$

and so on

In general the number of electrons N_n^- leaving an end-wall in the nth half-cycle is

$$N_n^- = [\delta(1-a)]^{n-1}. \tag{1}$$

The number of ions are given by the sums of geometric progressions. It can easily be shown that provided no ions are lost the number of ions N_n^+ present during the nth half-cycle is

$$N_n^+ = a\frac{1-[\delta(1-a)]^n}{1-\delta(1-a)}. \tag{2}$$

(b) *Least permissible value of the multiplication factor*

It has been shown that the starting of the discharge is primarily a wall process. Therefore he number of ions cannot exceed the number of electrons, at least in the early stages of the discharge. If n, the number of half-cycles, is large, and if the number of electrons, $[\delta(1-a)]^{n-1}$ is much greater than 1, from (1) and (2) the ratio of number of ions to electrons is

$$\frac{N_n^+}{N_n^-} \simeq \frac{a}{\delta(1-a)-1}. \tag{2a}$$

This should not exceed 1, which leads to the condition for the multiplication factor

$$\delta \geqslant \frac{1+a}{1-a} \approx 1+2a, \tag{2b}$$

which gives a minimum value for δ provided $a \ll 1$.

This minimum value of δ refers only to the starting of the discharge; δ may have other lower values during the development.

(c) *Growth of the wall charges*

Now the field due to the wall charges is $4\pi(\sigma_1-\sigma_2)$, and by evaluating the geometric progressions for σ_1, σ_2 from table 1 the field E_n due to the wall charges in the nth half-cycle is

$$E_n = 4\pi e \frac{(\delta-1)(1-a)}{1+\delta(1-a)} \{[\delta(1-a)]^{n-1} \pm 1\}$$

$$= 4\pi e \frac{(\delta-1)(1-a)}{1+\delta(1-a)} N_n^-, \quad \text{for large } N_n^-. \tag{3}$$

Thus if a, i.e. gas ionization, is small, and δ only a little greater than 1, then the wall-charge field E_n is proportional both the $(\delta-1)$ and to the number of electrons oscillating.

(d) *Effect of the wall charges on the motion of the electrons*

The equation of motion of an electron under the influence of an applied field $X \sin(\omega t+\phi)$ and a retarding field E due to wall charges is

$$m\frac{\mathrm{d}^2x}{\mathrm{d}t^2} = eX\sin(\omega t+\phi) - eE. \tag{4}$$

Integration, and insertion of the boundary conditions $\mathrm{d}x/\mathrm{d}t = v_0$ when $t = 0$ and $x = 0$ when $t = 0$, gives the velocity

$$\frac{\mathrm{d}x}{\mathrm{d}t} = v_0 + \frac{eX}{m\omega}[\cos\phi - \cos(\omega t+\phi)] - \frac{e}{m}Et, \tag{5}$$

and the distance travelled

$$x = v_0 t + \frac{eX}{m\omega}\left[t\cos\phi + \frac{1}{\omega}(\sin\phi - \sin[\omega t+\phi])\right] - \frac{eE}{2m}t^2. \tag{6}$$

This equation applies for any one transit across the tube. When $x = d$ the length of the tube, t is the transit time. It is more convenient to use the parameter ωt, which is the angular measure of the transit time. Let $x = d$ and write $v = eX/m\omega$. Then equation (6) becomes

$$\omega d = v_0(\omega t) + v\{\omega t\cos\phi + \sin\phi - \sin(\omega t+\phi)\} - \tfrac{1}{2}v\frac{E}{X}(\omega t)^2. \tag{7}$$

This is a transcendental equation and cannot rigorously be solved for ωt. However, if we assume that an electron takes very nearly half a cycle to cross the vessel, i.e. $\omega t \simeq \pi$, then in the above equation $\sin(\omega t+\phi) \simeq -\sin\phi$, and it becomes a quadratic in ωt. The solution for the transit time is

$$\omega t = \frac{\dfrac{\omega d}{v} - 2\sin\phi}{\cos\phi + \dfrac{v_0}{v}} + \frac{1}{2}\frac{E}{X}\frac{\left(\dfrac{\omega d}{v} - 2\sin\phi\right)^2}{\left(\cos\phi + \dfrac{v_0}{v}\right)^3}. \tag{8}$$

Two factors thus govern the transit time. One is represented by the first term, which for a given X and ω depends only on the initial velocity v_0 and the phase ϕ; the other depends chiefly on E/X, the ratio between the wall-charge field and the applied field. This ratio is always very small. The second term therefore gives the retardation due to the wall charges, and we shall now calculate its magnitude. Statistical variations can be neglected when the number of electrons oscillating in the vessel has grown to 50 or 100. From equation (3) the difference $\sigma_1 - \sigma_2$ in the wall charges is then about 10 or $20e$. The applied field X is about 20 V/cm when $\lambda = 18$ m. Now we choose any reasonable initial velocity v_0 for the secondary electrons (in the range 0·5 to 6 eV), and the corresponding phase in order that the transit time in the absence of wall charges should be exactly half a cycle. Then from equation (8), the increase $\delta(\omega t)$ in transit time due to this wall-charge field is

$$\delta(\omega t) = 5 \times 10^{-6} \text{ radian.}$$

Thus the secondary electrons released at the next impact will leave the end-wall, not in the phase ϕ but in the phase $\phi + \delta\phi_1$, where

$$\delta\phi_1 = \delta(\omega t) = 5 \times 10^{-6} \text{ radian.} \tag{9}$$

We assume throughout this treatment that there is no time lag between the arrival of a primary and the expulsion of a secondary electron. Measurements have shown that any lag which exists is less than about 10^{-12} s and therefore small compared with the period of the applied field.

We must now examine how the transit time in the following half-cycle depends upon this change in phase, as well as upon the wall-charge field. The dependence of transit time on the phase is given by $\partial(\omega t)/\partial\phi$. By differentiating equation (7)

$$\left(\frac{\partial(\omega t)}{\partial\phi}\right)_{v_0=\text{const.}} = \frac{\pi \sin\phi - 2\cos\phi}{v_0/v + 2\cos\phi} + \text{a very small term in } E/X. \tag{10}$$

Neglecting the correction term in E/X, this expression is zero when $\tan\phi = 2/\pi$ and is negative at smaller values of ϕ. It becomes increasingly negative when ϕ is negative. For a large range of wave-lengths ($\lambda \geqslant 17$ m) the numerical value of $\partial(\omega t)/\partial\phi$ is greater than 2, if we assume the initial energy of the electrons to be 2·5 eV or less. If the initial energy is greater than this then $|\partial(\omega t)/\partial\phi|$ is greater than 2 for a larger range of wave-lengths, including wave-lengths shorter than 17 m.

We assume now that the wall-charge field E first becomes effective at some half-cycle defined as number $M = 1$. During this half-cycle the retardation is due to the wall-charge field E only; in subsequent half-cycles the transit time is changed both by the wall-charge field and the different starting phase.

Let ϕ be the original phase and $\phi + \Delta\phi_m$ be the phase at the beginning of the half-cycle $M = m$. Let $\delta\phi_m$ be the increase in transit time during this mth half-cycle, due to the wall-charge field E_m alone.

Then the total change in transit time $\delta(\omega t)_m$ during the mth half-cycle is

$$\begin{aligned} \delta(\omega t)_m &= \delta(\omega t)_{\text{wall-charge field}} + \delta(\omega t)_{\text{phase}} \\ &= \delta\phi_m - \left|\frac{\partial(\omega t)}{\partial\phi}\right| \Delta\phi_m, \end{aligned}$$

remembering that $\partial(\omega t)/\partial\phi$ is negative. The phase at the beginning of the $(m+1)$th half-cycle (by definition $\phi+\Delta\phi_{m+1}$) is therefore equal to $\phi+\Delta\phi_m$, the phase at the beginning of the mth half-cycle, plus this change in transit time $\delta(\omega t)_m$. Hence

$$\phi+\Delta\phi_{m+1}=\phi+\Delta\phi_m+\delta\phi_m-\left|\frac{\partial(\omega t)}{\partial\phi}\right|\Delta\phi_m;$$

therefore $$\Delta\phi_{m+1}=\delta\phi_m-p\Delta\phi_m,$$

where $$p=\left|\frac{\partial(\omega t)}{\partial\phi}\right|-1.$$

Any $\Delta\phi_m$ is therefore given by a series in $\delta\phi_1, \delta\phi_2, \ldots, \delta\phi_{m-1}$. Since $\delta\phi_m$ is proportional to E_m, which is given by equation (3), then

$$\frac{\delta\phi_m}{\delta\phi_{m-1}}=\frac{E_m}{E_{m-1}}=\delta(1-a).$$

The series for $\Delta\phi_m$ therefore becomes a geometric progression leading to

$$\frac{\Delta\phi_m}{\delta\phi_1}=\pm p^{m-2}\left\{\frac{1\pm\left[\frac{1}{p}\delta(1-a)\right]^{m-1}}{1+\frac{1}{p}\delta(1-a)}\right\}. \tag{11}$$

$\Delta\phi_m/\delta\phi_1$ represents the total change in phase from the original phase ϕ in which the electron started, in terms of the phase change $\delta\phi_1$, in the first half-cycle during which the wall charge became effective (see equation (8) et seq.).

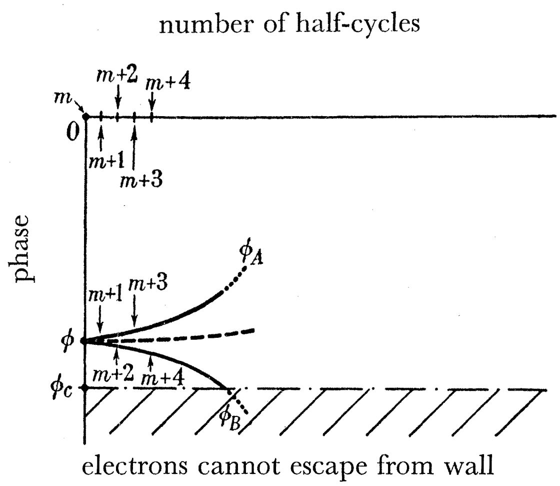

FIGURE 1. Effect of positive wall charges on the phase at which electrons leaves the walls. ϕ ... initial phase; ϕ_c–·–·, cut off phase; ϕ_A, ϕ_B, phase at the wall A and B respectively at which electrons arrive.

The general description of equation (11) is shown in figure 1. An electron leaving one wall in a phase ϕ is first retarded by the wall charge and arrives late at the opposite wall A (top curve). The secondary electrons produced are also retarded by the positive wall charge on A, but because they start in a phase more positive than ϕ they travel faster. The net result is that they arrive early at B (bottom curve).

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Assuming m is large and $\delta(1-a) \approx 1$, two limiting conditions follow: When $p > 1$, i.e. $|\partial(\omega t)/\partial\phi| > 2$, equation (11) becomes $\pm\frac{1}{2}p^{m-2}$. This is a divergent oscillation of the phase shown by the upper and lower curves in figure 1. When the lower curve crosses the critical phase line ϕ_c, the secondary electrons are driven back to the wall and the multiplication process will cease.

An example shows how many half-cycles will elapse until an electron drifts from the starting phase ϕ into the cut-off phase ϕ_c. If $\lambda = 18$ m then from (9) $\delta\phi_1 = 5\times10^{-6}$ and from figure 2, assuming $\epsilon_0 = 4$ eV, $\Delta\phi_m = \phi_c - \phi = -6°$. Also for the same conditions, (10) gives $\partial(\omega t)/\partial\phi$ and hence $p = 1{\cdot}1$. Equation (11) then gives $m \simeq 100$ half-cycles, at which time multiplication ceases.

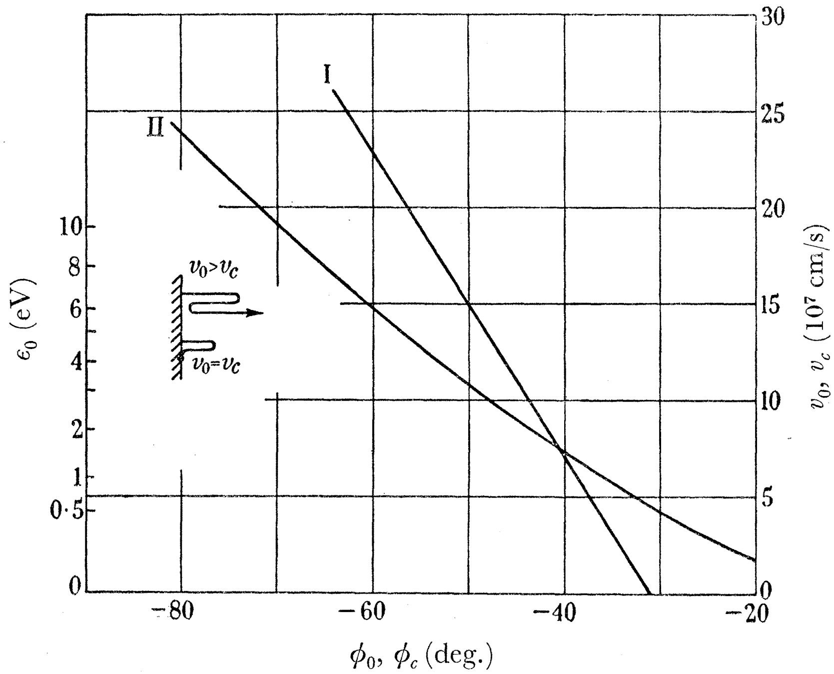

Figure 2. Curve I: initial velocity v_0 and energy ϵ_0 of electrons as a function of the phase ϕ_0, when the transit time is exactly half a cycle. Derived from the solution of equation (7), putting $\omega t = \pi$. Curve II: critical velocity v_c (least velocity necessary for an electron to escape from the wall) as a function of the phase ω_c. Derived from the solution of two simultaneous equations: $x = 0$ from equation (6), $dx/dt = 0$ from equation (5). Both curves calculated with $\lambda = 18$ m.

The treatment shows that at wave-lengths near the cut-off the discharge cannot start, which is contrary to experience. This must be due to a simplifying assumption on which the theory was based.

4. The multiplication process when secondary electrons have a distribution of initial velocities

(a) *Qualitative description*

The simplifying assumption of the last paragraph, namely, that secondary electrons all have a constant initial velocity v_0, is not likely to be true. In general, primary electrons of a single velocity which hit any solid produce secondaries having a range of velocities and we shall now extend the theory to include the effect of a velocity distribution. Very little is known about secondary emission from insulators, but from the published data (Kadyschevich 1945; Geyer 1942; Vudinski 1939) it appears that secondary electrons from glass should have a velocity distribution similar to that for metals, but with a slightly lower most probable velocity (McKay 1951). It is convenient to assume a velocity distribution

$$\mathrm{d}N = N_0 \cos^2 \frac{\pi}{12}(10-10^{-7}v)\,\mathrm{d}v. \qquad (12)$$

Expressed as an energy distribution, this corresponds to a curve stretching from 0·5 to 7 eV, with a peak at 2·5 eV, and of a similar shape to the known energy distributions for certain insulators.

Consider now a group of secondary electrons leaving one end-wall together. Their initial velocities are distributed; their transit times differ accordingly, and they hit the opposite wall at different instants. The new secondaries which they produce therefore start in different phases of the field. The first effect of the velocity distribution is therefore to create a distribution of the secondary electrons in phase. But the secondary electrons which start in any elementary phase interval $\mathrm{d}\phi$ have always the same velocity distribution given by equation (12). Hence there exist both a phase distribution and a velocity distribution among the secondary electrons.

For the secondary electrons to multiply it is necessary that

(i) the number of electrons must increase in each half-cycle, i.e. losses to the walls must not exceed a certain amount;

(ii) the distribution of secondary electrons in phase must be repeated in succeeding half-cycles; more strictly, the phase distribution in any half-cycle must at least include the distribution of the preceding half-cycle.

The second condition ensures that at no stage in the development do the majority of electrons become displaced in phase and lost, as described in the last section.

The two limiting conditions may be expressed formally thus:

$\mathrm{d}n/\mathrm{d}t > 0$, where n is the number of electrons present at any instant t;

$\mathrm{d}/\mathrm{d}t(F(\phi)\,\mathrm{d}\phi) = 0$, where $F(\phi)\,\mathrm{d}\phi$ describes only the shape and position of the phase distribution curve, but not its magnitude.

We shall now consider how the phase distribution varies throughout the multiplication process. If the transit time ωt is plotted against initial velocity v_0 for various phases ϕ (figure 3 evaluated from equation 7) an almost linear relation is found between ωt and v_0. Because of this linearity a group of electrons which start at the same instant (in a negative phase ϕ) but with initial velocities distributed according to a $\cos^2$ law produces secondaries which are distributed in phase according to a $\cos^2$ law. This assumes that all the electrons cross the vessel. But the slowest ones are driven back to the wall (see § 1) and are lost, and hence those secondary electrons which would fill up the positive end of the phase distribution are not created at all. The result is a distribution in phase of a $\cos^2$ shape chopped at its positive end.

It can be shown by the graphical method about to be described how these newly created electrons produce in the next few half-cycles a uniform distribution in phase. The sequence A, B, C, D in figure 4 shows how this uniform phase distribution is repeated in all subsequent half-cycles. For simplicity the $\cos^2$ velocity distribution is drawn as a triangle. From the initial phase distribution, figure 4*A*, the electrons (dn in number) leaving one wall in the phase interval ϕ to $\phi+\mathrm{d}\phi$ are selected.

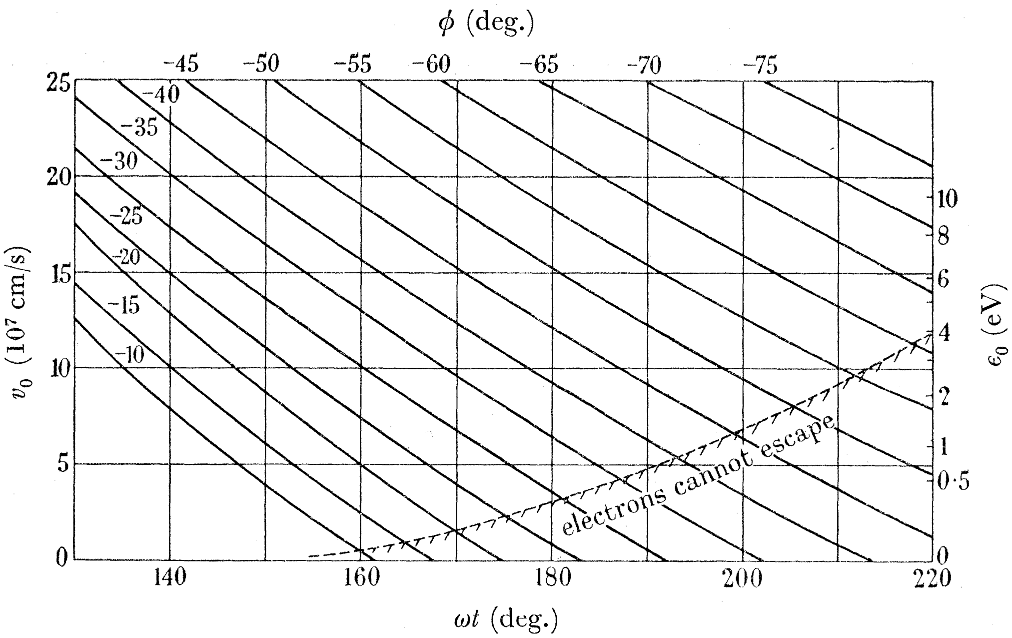

FIGURE 3. Initial velocity v_0, initial energy ϵ_0 and critical velocity v_c as a function of the transit time ωt for various phases ϕ, at $\lambda = 18$ m, calculated from equation (7) with $d = 6$ cm.

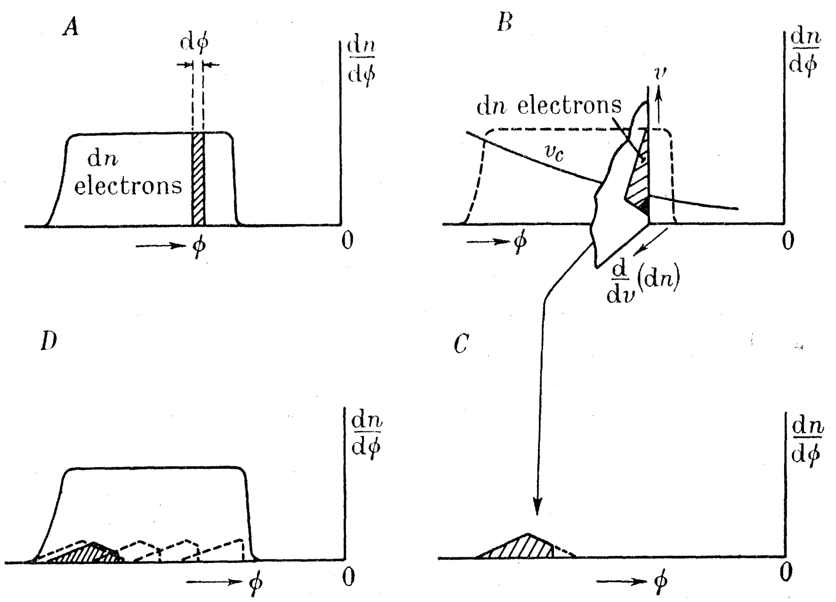

FIGURE 4. Maintenance of the phase distribution. *A* and *B*, uniform phase distribution in a given half-cycle at wall 1; *C* and *D*, phase distribution half a cycle later at wall 2; in *D* the distribution is the sum of a large number of chopped triangles. Ordinates of *C* and *D* are δ' times larger than those of *A* and *B*.

The triangle in figure 4*B* shows the velocity distribution of these dn electrons. The shaded areas are equal, because in both *A* and *B*, the area represents the number dn of electrons. The curved line in *B* represents the critical velocities v_c for various starting phases. Slower electrons are driven back to the wall.

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Figure 4*C* represents the secondary electrons at the beginning of the next half-cycle, produced by the electrons which crossed. The velocity distribution of the original electrons has caused a distribution in phase of the secondaries, as already described. The ordinate scale of picture *C* is greater than that of *A* by a factor δ' (the secondary emission coefficient) to account for the increase in number of the electrons due to secondary emission.

This process must be repeated for the electrons in every elementary phase interval $d\phi$ in the initial distribution. From each phase interval there result secondary electrons whose distribution in phase is shown by a triangle (chopped at one end) as in *D*. The resulting distribution in phase of all the secondaries is the sum of all such triangles, as shown in *D*. It is most notable that this also is a uniform distribution covering the same range of phase as the original one. This distribution in phase will include the initial distribution if the ordinate scale of *C* and *D* is large enough, i.e. if δ' is great enough. It must be remembered, however, that this construction takes account only of those losses caused by slow electrons being turned back to the wall, the 'cut-off' process. An accurate graphical analysis by the method just described shows that if $\delta' \approx 1{\cdot}05$ the distribution in *D* will just include the distribution in *A*. Physically this means that the 5% gained by multiplication is lost by the cut-off process, and so the number crossing the vessel would remain constant in each half-cycle. In addition to these losses, however, some electrons are lost to the side walls and hence a multiplication greater than 1·05 is necessary. Now from (5), using previous measurements of the starting field, the velocity of the electrons when they hit an end-wall has been calculated (85 eV) and for this energy Salow and Mueller measured $\delta' = 1{\cdot}4$. This value seems to be large enough to exceed losses both to the side walls and by cut-off.

An estimate of the losses to the side walls has been made, based on the known angular distribution of secondary electrons from metals. This shows that under the given experimental conditions (see later) about 50 % of the electrons hit the side walls. Moreover, not all of these are lost; some produce secondary electrons in favourable phases, and it has been shown by previous experiments at shorter wave-lengths by Gill & von Engel that these can make a large contribution to the multiplication. Also those which stick to the side walls produce a radial electric field which reduces further losses.

Taking these four effects into account it seems reasonable to assume that a value of $\delta' = 1{\cdot}4$ will just balance the losses. Briefly, $\delta' = 1{\cdot}4$ gives a multiplication factor δ just greater than 1.

(*b*) *Analytical treatment*

Suppose that in the *m*th half-cycle secondary electrons leave the wall distributed in phase according to the equation

$$dn_{\phi_1,\,\phi_1+d\phi} = n(\phi_1)\,d\phi,$$

and the electrons leaving in any phase interval $d\phi$ are distributed in velocity according to the equation

$$dN = f(v)\,dv.$$

The fraction g, having velocities in the range v, $v+dv$, is

$$g = \frac{dN}{N} = \frac{f(v)\,dv}{\int_{v_1}^{v_2} f(v)\,dv},$$

where v_1, v_2 are the limits of the velocity distribution. Now consider the number of secondary electrons $\mathrm{d}n'$ created in the $(m+1)$th half-cycle in the range ϕ, $\phi+\mathrm{d}\phi$. Of the $\mathrm{d}n$ that left the opposite wall in the phase interval ϕ_1, $\phi_1+\mathrm{d}\phi$, only a fraction g with the proper initial velocity create secondaries in the phase interval ϕ, $\phi+\mathrm{d}\phi$. The number of secondaries thus created is

$$\mathrm{d}(\mathrm{d}n') = \delta' g n(\phi_1)\,\mathrm{d}\phi = \delta' \frac{f(v)\,\mathrm{d}v}{N} n(\phi_1)\,\mathrm{d}\phi.$$

Hence the total number of secondaries in this phase interval ϕ, $\phi+\mathrm{d}\phi$ is

$$\mathrm{d}n' = \delta' \int_{\bar{v}_c}^{v_2} \frac{f(v)\,\mathrm{d}v}{N} n(\phi_1)\,\mathrm{d}\phi, \tag{13}$$

where $\bar{v}_c$ is the critical velocity averaged over the range of the original phase distribution. Physically, this means that electrons arriving in the phase interval $\mathrm{d}\phi$ must have the proper combinations of starting phase and initial velocity, provided that the velocity is not less than the critical value.

A numerical example can now be given. The distribution in velocity is assumed to be given by (12). Integration between velocities 4 to 16×10^7 cm/s ($\sim$ 0·5 to 7 eV) gives

$$N = 6\times10^7 N_0.$$

Since the first electrons to cross have a $\cos^2$ velocity distribution, the secondaries have a $\cos^2$ phase distribution of the form

$$\mathrm{d}n = n(\phi_1)\,\mathrm{d}\phi = n_0 \cos^2 k(\theta+\phi_1)\,\mathrm{d}\phi;$$

k is a constant which determines the spread of the distribution. The difference in transit time between the slowest and fastest electrons (figure 3) is about 40°, and the distribution in phase of the secondaries will spread over the same range, about a central angle, θ. Such a distribution is described by putting $k = \frac{1}{40}\pi$. Furthermore, there is a relation between ϕ_1, the phase in which the original group of electrons leaves the wall, and v, the initial velocity which causes secondary electrons to be released in the interval ϕ, $\phi+\mathrm{d}\phi$. This relation is

$$\phi_1 = -(P+Lv+\phi),$$

where P, L are constants ($P = 62{\cdot}5$, $L = 2{\cdot}5\times10^{-7}$, for $\lambda = 18$ m). The relation is derived from the graphical solution of equation (7) (see figure 3). The integration of equation (13) can now be performed. Neglecting for the moment losses caused by the slow electrons being turned back to the wall, and again assuming $f(v)\,\mathrm{d}v$ given by (12),

$$\mathrm{d}n' = \delta' \int_{v=4\times10^7}^{v=16\times10^7} \frac{f(v)\,\mathrm{d}v}{N} n(\phi_1)\,\mathrm{d}\phi$$
$$= \delta' n_0[0{\cdot}5 - A + 2A\cos^2 k(\phi+P+10^8 l-\theta)]\,\mathrm{d}\phi,$$

where A is a constant smaller than 0·5. This therefore represents a uniform phase distribution plus a cosine square distribution smaller than the original one. The peak occurs at the same phase as in the original distribution when

$$(\theta+\phi) = (\phi+P+10^8 l-\theta),$$

i.e. when $\theta = -41°$ for $\lambda = 18$ m.

It can easily be shown that if this calculation is repeated, using the above distribution as the initial one, the resulting distribution in phase is still more uniform. After five or six half-cycles a completely uniform phase distribution is obtained. The losses due to cut-off can now be considered by choosing the proper lower limit for the integration.

Assume a uniform initial phase distribution

$$\mathrm{d}n = n(\phi_1)\,\mathrm{d}\phi = n_0\,\mathrm{d}\phi.$$

The distribution in phase in the next half-cycle, considering the losses due to cut-off, is

$$\mathrm{d}n' = \delta' \int_{\bar{v}_c}^{v_2} \frac{f(v)\,\mathrm{d}v}{N}\, n_0\,\mathrm{d}\phi;$$

$\bar{v}_c$ over the range $\phi_1 = -20°$ to $-60°$ is given from figure 3 as 6×10^7 cm/s.

Then

$$\mathrm{d}n' = \delta' n_0\, 0{\cdot}97\,\mathrm{d}\phi.$$

This is again a uniform phase distribution which includes the initial distribution if

$$\delta' n_0\, 0{\cdot}97\,\mathrm{d}\phi \geqslant n_0\,\mathrm{d}\phi,$$

i.e. if

$$\delta' \geqslant 1{\cdot}03.$$

The analytical treatment thus confirms the result obtained graphically, namely, that a value of δ' about 1·05 is sufficient to maintain the number of electrons if they have any reasonable velocity distribution.

In fact the calculated velocity of impact of the electrons against the end-walls (85 eV) gives $\delta' \simeq 1{\cdot}4$. This larger value compensates for losses of electrons other than by the cut-off process, in particular, losses to the side walls because the secondary electrons are not all emitted normally from the end-walls.

(c) *Effect of the wall-charge field*

Since the secondary electrons are produced at different times within a certain phase range, the wall acquires its increment of positive charge at the same rate, and not instantaneously. The wall-charge field therefore varies as the electrons move across the tube, and only reaches its final steady value for a particular half-cycle when all the electrons have left the end-wall. Those electrons which leave first (i.e. in the most negative phase) leave few positive charges on the end-wall and are accelerated by the positive charge on the opposite wall. Thus their transit time is shorter than their negative starting would indicate. They produce secondaries at the opposite wall starting in a phase slightly more negative than might be expected.

Electrons which leave late (i.e. in the most positive phase) suffer the reverse effect. They are retarded by the full value of the wall-charge field, have therefore a longer transit time, and produce secondaries in a more positive phase than might be expected.

The effect of the positive-wall charges is to restrict the spreading of the phase distribution into more positive or more negative phases, and so help the development of the discharge.

5. The second stage of the discharge

The second stage of the discharge begins when the space charge due to positive ions first affects the motion of the electrons, thereby changing the energy of impact at the walls by a small percentage. This stage does not always exist; at very low pressure, self-repulsion of the electrons becomes a dominant factor before a large number of ions have been produced. This effect is discussed later.

(a) *Distribution of ions in space*

During the first stage, electrons cross the tube continually from one end to the other, leaving one wall with a small energy, about 0·5 to 7 eV, and hitting the opposite wall with an energy of about 90 eV. During every transit each electron has a certain chance of ionizing a gas molecule. This chance depends upon pressure and the nature of the gas, and

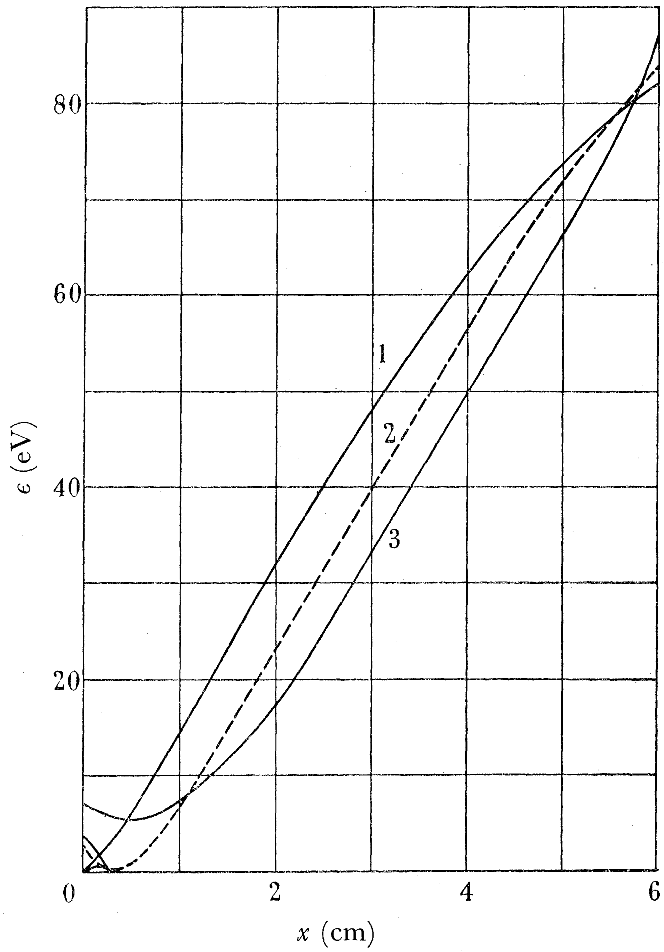

FIGURE 5. Energy ϵ of the electrons as a function of their distance x from the wall, for various initial velocities v_0 and associated phases ϕ (calculated with $\lambda = 18$ m). Wall and space charges absent. Curve 1, $\phi_c = -60°$, $v_0 = v_c = 1\cdot15 \times 10^8$ (cm/s); curve 2, $\phi = -40°$, $v_0 = 10^8$ cm/s; (2·8 eV); curve 3, $\phi = -20°$, $v_0 = 1\cdot6 \times 10^8$ cm/s (7 eV).

upon the energy of the electron, being greatest where the electron is fastest. It would seem therefore that two concentrations of positive ions would be created near the end-walls, where the electrons are fastest. Detailed examination shows that this is not so (Francis & von Engel 1950).

From equations (5) and (6) one can calculate how the energy of an electron varies with its distance along the vessel, in the absence of space charges. The result is shown in figure 5

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for the extreme conditions, namely, the fastest and slowest electrons of the velocity distribution. The fastest electrons are those which start in the most positive phase with the greatest initial velocity; the slowest are those which start in the most negative phase with an initial velocity just equal to the critical velocity v_c. The ionization produced is calculated by combining the energy-distance curve with the known ionization probabilities for various electron energies and various gases (von Engel & Steenbeck 1932), and this is plotted in

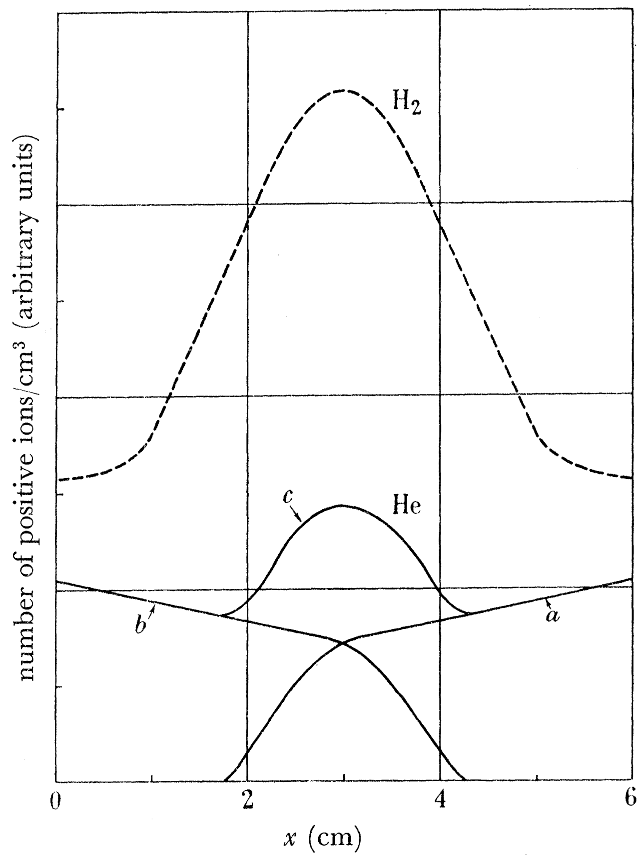


FIGURE 6. Average distribution of positive ions in space at the beginning of the second stage in helium and hydrogen (n_+ ... number of positive ions/cm^3). Curve *a*, ionization produced by an electron moving from the left to the right wall and curve *b*, when it is moving from right to left; curve *c*, total ionization.

figure 6; (*a*) shows the ionization produced in helium by electrons moving across the tube in one direction; (*b*) shows the ionization produced in the next half-cycle by electrons moving in the opposite direction. The resultant distribution of helium ions in space is the sum of the two curves (*c*), and it is found to have a maximum at the middle of the tube, more or less pronounced depending on the gas. These ions can be regarded as stationary; they move with only thermal velocity and take several hundred half-cycles of the applied field to reach the walls.

To calculate the effect which the ions have on the motion of the electrons a uniform distribution of ions will be assumed for simplicity. This is reasonable, since even in hydrogen the concentration of ions at the centre is only twice that at the walls (figure 6), whilst in helium the difference is less marked.

(b) Effect of the ion space charge on the motion of the electrons

During the second stage of the discharge electrons move across the tube under the influence of a field made up of several components—the applied field, the wall-charge field and the field due to positive ions distributed in the vessel. Two simplifying assumptions will be made:

(i) the positive ions are distributed uniformly throughout the vessel, n_+ ions per cm^3;

(ii) the electrons move in a thin layer across the vessel.

From equation (6) it can be shown that the fastest and the slowest electron crossing the vessel (in the absence of space charge and provided that there is no appreciable self-repulsion among the electrons) are never farther apart than 1 cm, and most of the time much less than that. We may therefore reasonably assume that the bulk of electrons move in a thin layer.

Consider the plane of electrons to be x cm from one end-wall at a time t and moving towards the opposite wall (figure 7). Then applying Gauss's theorem to the region $ABCD$, and considering only the field due to the charges in the vessel, ignoring the applied field, we have

$$E_{\text{ions}} = 4\pi\{\sigma_{\text{wall}} + n_+ ex - \tfrac{1}{2}\sigma_-\},$$

where σ_- is the charge per sq.cm in the plane of electrons and n_+ the number of ions per cm^3.

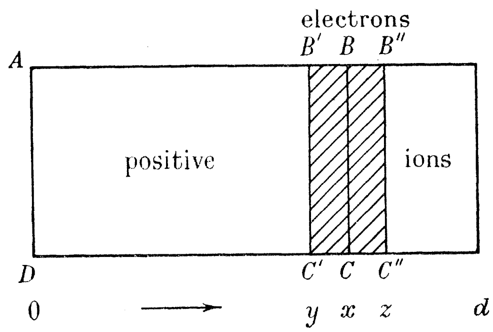


FIGURE 7. Motion of electrons in the second stage. Positive ions assumed uniformly distributed throughout the vessel.

The net charge in the vessel is, however, always zero, since no charge can flow from the external circuit into the glass vessel. Thus, when d is the length of the vessel

$$2\sigma_{\text{wall}} + n_+ cd - \sigma_- = 0.$$

Hence
$$E_{\text{ions}} = 4\pi n_+ e(x - \tfrac{1}{2}d). \tag{14}$$

The total force on the electrons resulting from the applied field $X \sin(\omega t+\phi)$ minus the field due to the ions, leads to the equation of motion for one half-cycle:

$$\frac{d^2x}{dt^2} + k^2x = \frac{eX}{m}\sin(\omega t+\phi) + K, \tag{15}$$

where
$$k^2 = \frac{4\pi n_+ e^2}{m} \quad \text{and} \quad K = \tfrac{1}{2}k^2 d.$$

k is the natural frequency of oscillation of the plane of electrons under the influence of the ions alone. For the boundary conditions $x = 0$ when $t = 0$, and $dx/dt = v_0$ when $t = 0$, we have

$$x = \frac{1}{k}(v_0 - \omega p\cos\phi)\sin kt - (p\sin\phi + \tfrac{1}{2}d)\cos kt + p\sin(\omega t+\phi) + \tfrac{1}{2}d, \tag{16}$$

where $p = \dfrac{eX}{m(k^2-\omega^2)}$ and provided $k \neq \omega$. For $k^2 = \omega^2$ another finite solution can be obtained. The velocity is given by

$$\frac{dx}{dt} = (v_0 - \omega p \cos\phi)\cos kt + k(p\sin\phi + \tfrac{1}{2}d)\sin kt + \omega p\cos(\omega t+\phi). \tag{17}$$

Convenient parameters are k/ω and ωt. Since $k^2 \propto n_+$ a variation of k/ω corresponds to altering the number of ions in the vessel. The result is shown in figure 8 and will now be summarized.

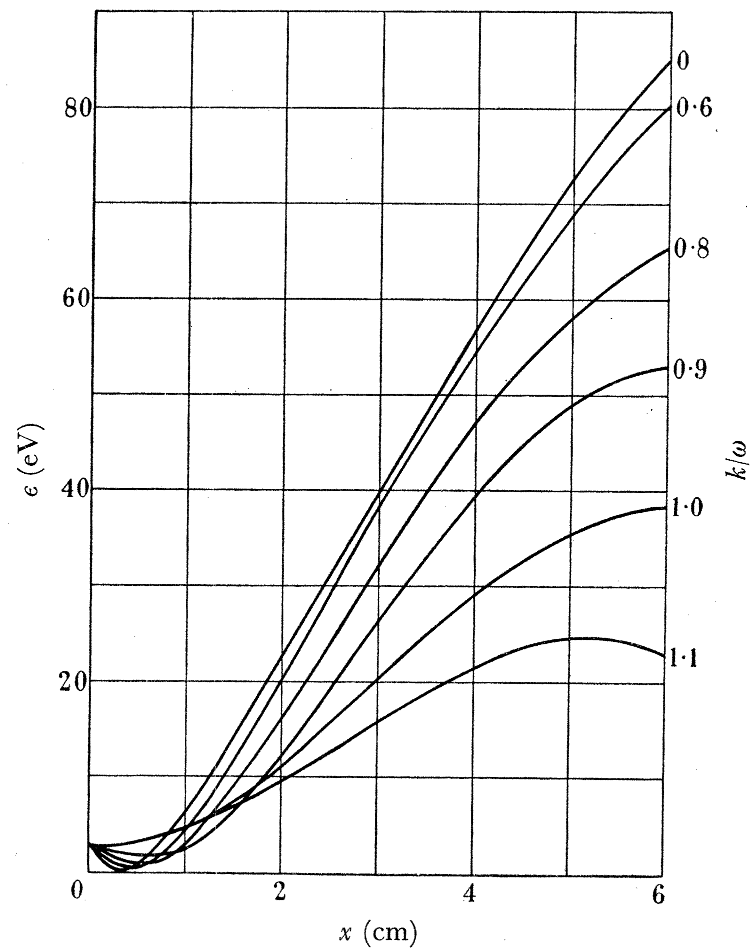


FIGURE 8. Energy ϵ of the electrons as a function of their distance x from an end-wall during the second stage for various concentrations n_+ of positive ions (derived from equations (16) and (17)); $k/\omega \propto \sqrt{n_+}$.

As the number of ions in the vessel increases,

(α) the velocity of the electrons in flight and at impact steadily decreases. The production of ions in space and of secondary electrons at the walls therefore decreases;

(β) the transit time decreases and therefore secondary electrons are released in more negative phases;

(γ) secondary electrons are pulled quickly away from the wall by the ions; losses by the cut-off process are reduced.

From figure 8 one can derive, by the method explained for figures 5 and 6, the number of ions (a) produced by each electron in a single transit. Also from the energy of impact one can find the secondary emission coefficient δ' for various space-charge densities. These results are plotted in figure 9.

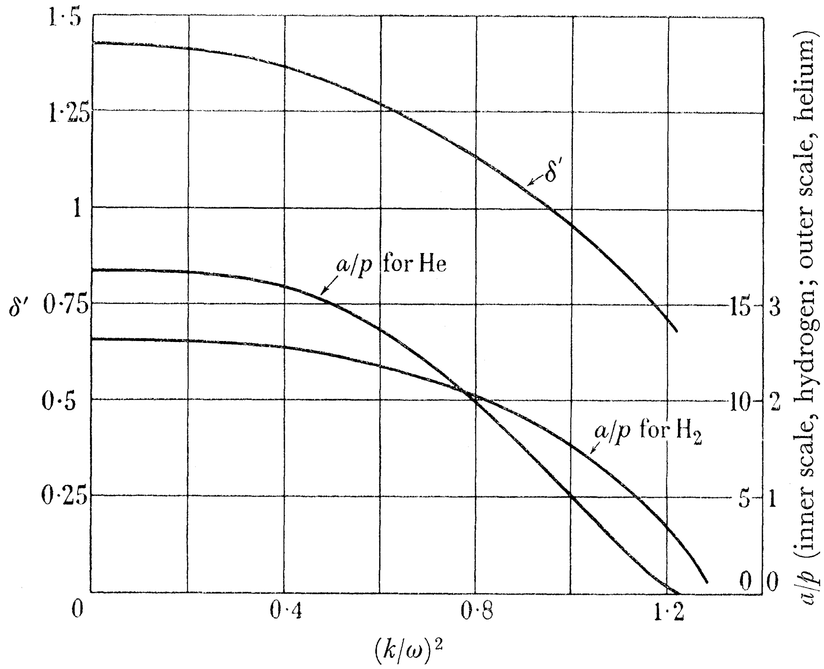


FIGURE 9. Variation of the secondary emission coefficient δ', and the number of ions a, produced per half-cycle by one electron, with the concentration n_+ of positive ions in the gas $((k/\omega)^2 \propto n_+)$.

From (α) it is clear that during the second stage the current increases more slowly than if the mechanism in the first stage were to persist, and may be described by a function

$$i \propto \delta^{\alpha t} f(t),$$

where $f(t)$ is a steadily decreasing function of the time, but is unlikely to be any simple function.

Except when the positive ion space charge becomes large (at the end of the second stage) the few electrons which collide with and ionize gas molecules are assumed to be lost, since they fall out of step with the main body.

6. THE RATE OF GROWTH OF CURRENT

(a) *Calculation of the current*

Suppose that the cylindrical vessel of length d is placed between plane parallel electrodes a distance l apart, which having a potential V between them produce the uniform field X applied. Consider a charge q which moves a distance dx in the field in a time dt (von Engel & Steenbeck 1932). An instantaneous current i flows in the external circuit connected to the electrodes. By the conservation of energy

$$qX\,dx = iV\,dt$$

$$i = q(X/V)\,dx/dt = (q/l) \times \text{velocity of charge.}$$

In any half-cycle if the charge q consists of N_- electrons

$$i_{\text{peak}} = (eN_-/l) \times \text{max. velocity.} \tag{18}$$

In our experiments $d = 6$ cm and $l = 7$ cm.

(*b*) *The first stage*

We have shown that in order to overcome losses of electrons by the cut-off mechanism the secondary emission coefficient δ' must be greater than 1·03. This figure depends upon the velocity distribution assumed, but it is not likely to be much greater than 1. The actual value of δ' obtained from measurements of the starting potential is, however, about 1·4 (when $\lambda = 18$ m) corresponding to an energy of impact of 85 eV. This larger value of δ' is necessary to overcome losses from various causes, e.g. angular distribution of secondary electrons causing losses to the side walls; scattering of electrons to the side walls by elastic collisions with gas molecules; surface effects on the glass caused by adsorbed glass layers.

Losses to the side walls will decrease as the positive space charge grows in the vessel, tending to pull the electrons into the centre. Adsorbed gas is likely to be of less importance in the later stage of the discharge when the glass is bombarded by large numbers of electrons.

Positive ions begin to affect the motion of the electrons when $(k/\omega)^2 \sim 0{\cdot}2$ (see figure 9). This is the point where the natural frequency of the electrons oscillating in a uniform positive space charge becomes comparable with the applied frequency. The first factor to be affected is the energy of impact and hence δ', while the ionization per half-cycle (a) remains constant in the range $(k/\omega)^2 = 0$ to 0·2. Now losses to the side walls depend—like the ionization—on the energy of the electrons which is a function of their position in the vessel, and so it is reasonable to assume that in this range the losses of electrons also remain constant (see figure 9).

When $(k/\omega)^2 > 0{\cdot}2$ we can only assume that losses of electrons decrease (like a) in some steady manner. The simplest assumption is that δ decreases to 1 according to the relation

$$\delta - 1 \propto \delta' - 1.$$

The first stage of the discharge is now defined by $(k/\omega)^2 < 0{\cdot}2$, i.e. when there are too few positive ions to affect the motion of the electrons apart from a very small decrease in δ'. The value $(k/\omega)^2 = 0{\cdot}2$ marks the beginning of the second stage during which the growing positive space charge exerts an increasing influence on the motion of the electrons.

(α) The first stage, in helium, at pressure 12×10^{-3} mm Hg:

When starting potential is applied, δ has initially its minimum value, namely, $\dfrac{1+a}{1-a}$ (equation (2*b*)). From figure 9, $a = 4 \times 10^{-2}$; hence $\delta = 1{\cdot}084$. During the first stage δ' decreases by 0·013 when $(k/\omega)^2$ reaches 0·2. Since the losses remain constant, δ decreases by the same amount. Hence, at the end of the first stage, $\delta = 1{\cdot}071$.

Thus during the first stage the initial conditions are:

$$a = 4 \times 10^{-2}, \qquad \delta = 1{\cdot}084, \qquad \delta(1-a) = 1{\cdot}04,$$
$$(k/\omega)^2 = 0, \qquad n_+ = 0, \qquad N_+ = 0,$$

where N_+ is the total number of ions in the vessel.

The conditions at the end of the first stage are

$$a = 4\times10^{-2},\qquad \delta = 1{\cdot}071,\qquad \delta(1-a) = 1{\cdot}028,$$
$$(k/\omega)^2 = 0{\cdot}2,\qquad n_+ = 6{\cdot}9\times10^5 \text{ from (15)},\qquad N_+ = 2{\cdot}07\times10^7.$$

assuming a vessel of length $d = 6$ cm, area of cross-section $= 5$ sq.cm. The growth of the number of ions is described by equation (2), and the number of oscillating electrons produced during the same period is given by equation (1). However, since $\delta(1-a)$ is not a constant throughout the first stage, an average value must be used. It is reasonable to take the geometric mean of the initial and final values of $\delta(1-a)$.

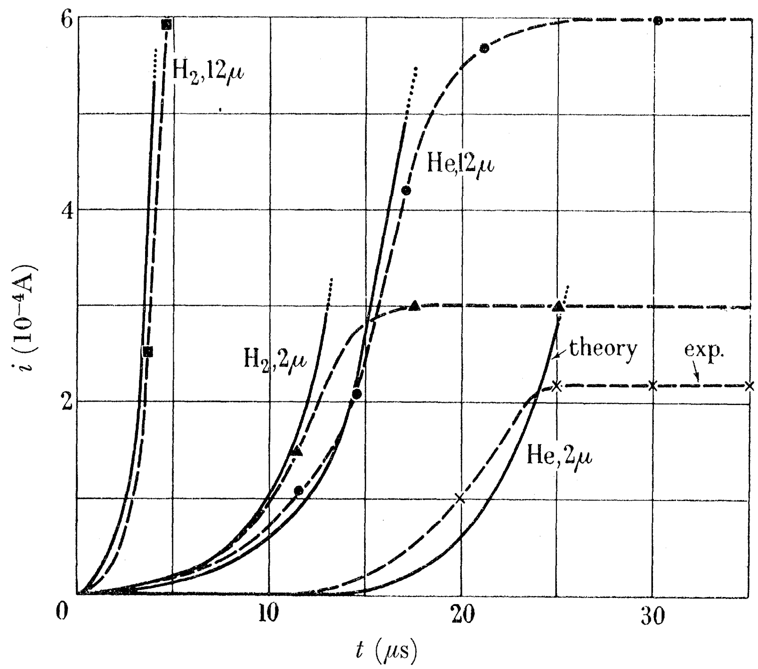


FIGURE 10. Rate of growth of current in helium and hydrogen at two different pressures from theory and experiments.

For helium, at a pressure of 12μ

$$\overline{\delta(1-a)} = \surd(1{\cdot}04\times1{\cdot}028) = 1{\cdot}034.$$

From (2) we get: number of half-cycles $n = 480$, corresponding at $\lambda = 18$ m, to a time $t_1 = 15{\cdot}5\,\mu$s. And from (1), number of electrons oscillating, $N_- = 1{\cdot}75\times10^7$. When $(k/\omega)^2 = 0{\cdot}2$ the maximum velocity occurs when the electrons hit the wall, and is

$$55{\cdot}3\times10^7\,\text{cm/s}\quad(83\,\text{eV}).$$

Inserting these values in equation (18) with $l = 7$ cm gives the current at the end of the first stage: after $t_1 = 14{\cdot}5\,\mu$s $i_1 = 2{\cdot}2\times10^{-4}$ A.

The growth of current in helium at pressure of 12μ, during the first stage of the discharge, is shown in figure 10.

(β) An exactly similar calculation for hydrogen at $p = 12\mu$ gives: initially

$$a = 15{\cdot}7\times10^{-2},\qquad \delta = 1{\cdot}373,\qquad \delta(1-a) = 1{\cdot}157,$$
$$(k/\omega)^2 = 0,\qquad n_+ = 0,\qquad N_+ = 0.$$

At the end of the first stage

$$a = 15{\cdot}7\times 10^{-2},\qquad \delta = 1{\cdot}36,\qquad \delta(1-a) = 1{\cdot}146,$$

$$(k/\omega)^2 = 0{\cdot}2,\qquad n_+ = 6{\cdot}9\times 10^5,\qquad N_+ = 2{\cdot}07\times 10^7.$$

Hence $$\overline{\delta(1-a)} = \sqrt{(1{\cdot}157\times 1{\cdot}146)} = 1{\cdot}152.$$

From (2) the number of half-cycles to produce $2{\cdot}07\times 10^7$ ions is

$$n = 120,\quad t_1 = 3{\cdot}5\,\mu\text{s},\quad \text{at}\quad \lambda = 18\,\text{m}.$$

From (1) the number of electrons $N_- = 2\times 10^7$, hence from (16)

$$i_1 = 2{\cdot}5\times 10^{-4}\,\text{A}.$$

This growth of current is also shown in figure 10.

(c) *The second stage*

During this stage, production of electrons by gas ionization and by secondary emission becomes of approximately equal importance. Self-repulsion of both ions and electrons also becomes important, and all these processes are so interdependent that an accurate calculation is very involved. What follows is an approximate treatment based on several criteria which, although somewhat arbitrary, are nevertheless derived from reasonable physical assumptions.

The range of $(k/\omega)^2$ which marks the second stage of the discharge is somewhat arbitrary. We have already defined it as beginning at $(k/\omega)^2 = 0{\cdot}2$ when the positive ions first have any noticeable effect. The treatment of the motion given in (15) is valid only when the layer of electrons remains reasonably thin, and when the distribution of ions is approximately uniform. As the number of electrons increases the layer widens, owing to the self-repulsive forces, and of course it widens during any one transit across the vessel. The motion of the electrons which travel at the extreme edges of the layer can be investigated on the simple assumption that the layer is very thin when it starts from an end-wall, so that the boundary condition is the same for all electrons. Application of Gauss's theorem to the regions $AB'C'D$, $AB''C''D$ (figure 7) then gives the fields at the edges of the layer. The equations of motion are then

$$\left.\begin{aligned}\ddot{z}+k^2z &= (eX/m)\sin(\omega t+\phi)+K'\\ \ddot{y}+k^2y &= (eX/m)\sin(\omega t+\phi)+K'',\end{aligned}\right\}\tag{19}$$

where y, z are the co-ordinates at the rear and front edges of the layer respectively and

$$K'/k^2 = (1+N_-/N_+)\,d/2,$$
$$K''/k^2 = (1-N_-/N_+)\,d/2.$$

Solution gives

$$\left.\begin{aligned}\left.\begin{matrix}z\\y\end{matrix}\right\} &= x\pm(N_-/N_+)\,(\tfrac{1}{2}d)\,(1-\cos kt),\\ \left.\begin{matrix}v_z\\v_y\end{matrix}\right\} &= v_x\pm(N_-/N_+)\,(\tfrac{1}{2}d)\,k\sin kt.\end{aligned}\right\}\tag{20}$$

The factor $(N_-/N_+)\,d/2\,(1-\cos kt)$ is the half-width of the layer, and depends both on the number of ions present ($\cos kt$ term, depending on N_+), and also on the ratio (N_-/N_+) of the number of electrons oscillating to the number of the ions in the gas. Equations (1) and (2) for the growth of electrons and ions are valid only when the layer of electrons is not too wide, i.e. when the above-mentioned factor does not exceed a certain value. Some arbitrary condition for the greatest width of the layer must be imposed: let us assume that when the middle of the layer hits the opposite wall the rear end of the layer should be at least halfway across the vessel, i.e. the half-width of the layer should not exceed half the length of the vessel. This gives the condition for (1) and (2) to be valid:

$$(N_-/N_+)\,(1-\cos kt) = N_-/N_+(1-\cos k\pi/\omega), \tag{21}$$

since $\omega t \approx \pi$ when $x = d$.

Self-repulsion of electrons affects slightly the distribution of ions in space. Ionization is produced mostly by the electrons travelling near the edges of the layer, which get enough energy to ionize before they are half-way across the vessel. Thus a positive space charge grows with an increasing concentration at the centre (see figure 6). This will be more marked in hydrogen, which has a lower ionization potential and a larger ionization probability than helium. Figure 8 does not show where in the vessel ionization occurs, although it does give an approximate value for the total ionization produced.

As the number of ions increases, losses of ions to the wall will increase owing to the field which now exists between the space charges and the wall charges. Since the net charge in the vessel is zero, all the lines of force from positive ions must end on the electrons, either oscillating or on the wall. Only the latter produce a steady field causing losses of ions which can thus be regarded as being due either to the attraction of the wall charges or to the self-repulsion of those ions not neutralized by the oscillating electrons. These losses may be calculated thus:

Consider a uniform sheet of positive ions of mass M and of concentration n_0 ions/cm^3, and of width d, whose lines of force end on two planes of negative charge each of n_- electrons per sq.cm.

The field at a distance x from the centre of the sheet is, by Gauss, $4\pi n_0 ex$, and the equation of motion of an ion of mass M in this position is

$$M\ddot{x} = 4\pi n_0 e^2 x. \tag{22}$$

The solution of this gives the velocity at any point, and hence at the edge ($x = \frac{1}{2}d$), from which the distance moved by the extreme ions in a time t (1 half-cycle) can be found. Knowing that $n_0 d = 2n_-$ we get that the total number of ions moving across each sq.cm of each boundary ($x = \pm\frac{1}{2}d$) of the sheet in a time t is

$$\{2\sqrt{(4\pi e^2/M)}\}\,(2n_-/d)^{\frac{3}{2}}\,(\tfrac{1}{2}d)\,t.$$

Applying this to our vessel, assuming an area of cross-section of 5 sq.cm, $d = 6$ cm, $t = 1$ half-cycle at $\lambda = 18$ m, and since by the conservation of charge $2n_-$ area $= N_+ - N_-$, the number of ions lost to both end-walls per half-cycle is

$$4{\cdot}9 \times 10^{-6}(N_+ - N_-)^{\frac{3}{2}} \quad \text{for} \quad H_2^+ \text{ ions},$$

or

$$3{\cdot}46 \times 10^{-6}(N_+ - N_-)^{\frac{3}{2}} \quad \text{for} \quad He^+ \text{ ions}.$$

To obtain this loss in the same notation as the rate of production a, we have the number of ions lost per half-cycle per oscillating electron

$$a' = \left.\begin{matrix} 4\cdot 9 \times 10^{-6} \\ 3\cdot 46 \times 10^{-6} \end{matrix}\right\} (N_+ - N_-)^{\frac{3}{2}}/N_- \quad \left.\begin{matrix} \text{for } H_2^+ \text{ ions} \\ \text{for } He^+ \text{ ions} \end{matrix}\right\}. \tag{23}$$

Equations (1) and (2) must now be modified to give the multiplication of electrons and the growth in the number of ions.

Consider the stage defined by the range $(k/\omega)^2 = 0\cdot 2$ to $0\cdot 6$. Assume that the corresponding increase in the number of ions, ΔN_+, takes m_1 half-cycles, then

$$\Delta N_+ = N_{-\text{initial}}\overline{(a-a')} \left\{ \frac{[\overline{\delta(1-a)}]^{m_1} - 1}{\overline{\delta(1-a)} - 1} \right\}. \tag{24}$$

As before $\overline{\delta(1-a)}$ is the geometric mean of the initial and final values, i.e. at $(k/\omega)^2 = 0\cdot 2$ and $0\cdot 6$, but $\overline{a-a'}$ is the arithmetic mean, since the growth of ions is an additive process.

Likewise, the growth of electrons is

$$(N_-)_{\text{final}} = (N_-)_{\text{initial}}[\overline{\delta(1-a)}]^{m_1}. \tag{25}$$

These equations are only valid provided that the final value of $\delta(1-a)$ in any range considered is not less than 1, and provided equation (21) is satisfied.

Enough data are now available to calculate the growth of electrons and ions in the range from $(k/\omega)^2 = 0\cdot 2$ until $\delta(1-a) = 1$, which occurs for hydrogen and helium when $(k/\omega)^2$ is between $0\cdot 6$ and $0\cdot 8$ by using equations (24) and (25). The following procedure is necessary. The value of a' at the beginning of the stage ($(k/\omega)^2 = 0\cdot 2$) can be calculated from (23); the value of a' at the end of the stage must be guessed. The values of a are given in figure 9, and thus the mean $\overline{(a-a')}$ is obtained. The mean $\overline{\delta(1-a)}$ can be obtained by using the initial and final values from figure 9, assuming that δ decreases to 1 according to the relation $(\delta - 1) \propto (\delta' - 1)$. Then from (24) the number of half-cycles m_1 and hence the duration of the stage is obtained. Equation (25) gives $(N_-)_{\text{final}}$, and equation (23) gives a' at the end of the stage. This calculated value of a' must now be compared with the value assumed. If there is any discrepancy a new value of a' must be assumed and the calculation repeated until the two values agree.

The current is then given by (18), using for the velocity of the electrons the value given in figure 8.

The calculations are given below for helium and hydrogen at a pressure of 12μ.

(α) *Helium at $p = 12\mu$*

Initial conditions, i.e. when $(k/\omega)^2 = 0\cdot 2$, $N_+ = 2\cdot 07 \times 10^7$, $\delta = 1\cdot 071$, $a = 4\cdot 01 \times 10^{-2}$, $\delta(1-a) = 1\cdot 028$, a' is negligible, $N_- = 1\cdot 75 \times 10^7$ electrons.

Final conditions, when $\delta(1-a) = 1$. This occurs when $(k/\omega)^2 = 0\cdot 78$, $N_+ = 8\cdot 06 \times 10^7$. Then $a = 2\cdot 52 \times 10^{-2}$; a' is assumed $= 1\cdot 3 \times 10^{-2}$. These give $\overline{\delta(1-a)} = 1\cdot 0139$ and $\overline{a-a'} = 2\cdot 62 \times 10^{-2}$. Equations (24) and (25) give: the duration of the second stage, $m_1 \simeq 76$ half-cycles $= 2\cdot 5\,\mu$s at $\lambda = 18$ m. The number of electrons oscillating, $N_- = 4\cdot 9 \times 10^7$ electrons.

Since from figure 8 the maximum energy of the electrons is 54 eV, the current from equation (18) is

$$i_2 = 4\cdot 8 \times 10^{-4}\,\text{A}.$$

a' calculated from (23) for this final state is then $1\cdot25\times10^{-2}$, consistent with the assumption. Inserting these values in equation (21) shows that $N_-/N_+(1-\cos(k/\omega)\pi) = 1\cdot17$. We have therefore only just exceeded the limit of validity of the treatment but it can be regarded as approximately correct.

The growth of current is plotted in figure 10.

(β) *Hydrogen at* $p = 12\mu$

Initial conditions, when $(k/\omega)^2 = 0\cdot2$, $N_+ = 2\cdot07\times10^7$, $\delta = 1\cdot360$, $a = 15\cdot72\times10^{-2}$, $\delta(1-a) = 1\cdot146$, a' is negligible, $N_- = 2\times10^7$ electrons.

Final conditions, when $\delta(1-a) = 1$, which occurs when $(k/\omega)^2 = 0\cdot72$, $N_+ = 7\cdot45\times10^7$.

Then $a = 13\cdot2\times10^{-2}$; a' is assumed $= 1\cdot4\times10^{-2}$. These give $\overline{\delta(1-a)} = 1\cdot0705$ and $\overline{a-a'} = 13\cdot75\times10^{-2}$. Equations (24) and (25) give

$$m \simeq 13 \text{ half-cycles}$$

Duration of stage $t = 0\cdot5\,\mu$s at $\lambda = 18$ m.

$$N_- = 4\cdot76\times10^7 \text{ electrons.}$$

The maximum energy is 60 eV. Hence, from (18) the current is

$$i_2 = 5\times10^{-4}\,\text{A}.$$

a' calculated from (23) for this final state is $1\cdot4\times10^{-2}$, agreeing with that assumed. When the above values are inserted, the factor in equation (21) becomes 1·2; thus we have again just exceeded the limit of validity of this treatment but the result will be approximately correct.

(d) *Further development of the discharge*

The treatment applied so far becomes invalid when the self-repulsion of the layer of electrons causes it to spread along the length of the vessel. The motion then resembles more a plasma oscillation. Ions and electrons are then formed predominantly in the gas. Owing to the increasing concentration of space charges losses of both ions and electrons increase. The distribution of ions in space also affects considerably the further growth of current. For example, a space charge which is mostly concentrated in the centre of the vessel is more effective than a uniformly distributed one in preventing losses of electrons to the walls.

For these reasons we expect that the rate of growth and the final current will be greater in hydrogen than in helium. First, the probability of ionization and hence the rate of production is greater in hydrogen, and secondly, hydrogen because of its lower ionization potential will have a greater concentration of ions near the middle of the vessel.

7. Development of the discharge at very low pressure

(a) *Qualitative treatment*

When the pressure is very low (e.g. 1 or 2μ or less) the gas ionization a is very small and the minimum value for δ (equation (2a)) is very little greater than 1. It would therefore take several thousand half-cycles before the electrons could multiply to any appreciable number. Although such a discharge is theoretically possible the smallest fluctuations in δ, especially in the early stages would stop the development. Hence, the chances of observing experimentally a discharge growing so slowly are almost zero. In order to obtain a discharge which can easily be observed, that is, one which has a reasonable probability of development, the applied field must be slightly increased, so that δ exceeds the minimum value given by (2*a*).

Now figure 5 shows that with the assumed velocity distribution the secondary electrons hit the end-walls with an energy between 83 and 87 eV, having an average of 85 eV. We have previously assumed that this energy of impact (85 eV) gives the correct minimum δ; but the secondary released by the impact may have an initial velocity such that it hits the opposite wall with an energy of only 83 eV which would give too low a δ to sustain the development. This difficulty can be overcome by assuming a slightly increased starting field such that the average energy of impact is increased, and the least energy of impact is now 85 eV instead of 83 eV. This corresponds to an increase in δ' and hence in δ (derived from the slope of the known curve of δ against energy) of approximately 1·5%, and is sufficient to ensure a multiplication of the electrons in spite of the fluctuations in δ. Thus the new condition at very low pressures is that

$$\delta_{\text{min.}} = \frac{1+a}{1-a} + 1\cdot5\ \%.$$

Using this value of δ in equations (1) and (2) shows that there are far more electrons oscillating than positive ions in the gas. Thus the growth of the discharge will finally be checked by self-repulsion of the electrons rather than by the space charge of the positive ions.

(*b*) *Quantitative examples*

(i) *Growth of current in helium at pressure* 2μ

Initial conditions:

$$a = 6\cdot7\times10^{-3}, \qquad \delta = \frac{1+a}{1-a} + 1\cdot5\ \% = 1\cdot028, \qquad \delta(1-a) = 1\cdot021,$$

$$N_- = 0, \qquad n_+ = 0, \qquad N_+ = 0.$$

It will soon become obvious that the final condition, i.e. the limit of validity of this treatment, is given at these low pressures by equation (21).

Final conditions:

$$(N_-/N_+)\,(1-\cos k\pi/\omega) = 1.$$

Now, from equation (2*a*),

$$N_-/N_+ = \frac{21\times10^{-3}}{6\cdot7\times10^{-3}} = 3\cdot14. \tag{26}$$

Hence
$$k/\omega = 0\cdot26, \quad (k/\omega)^2 = 0\cdot068,$$
$$n_+ = 2\cdot34\times10^5 \text{ from (15)}, \quad N_+ = 7\cdot04\times10^6.$$
$$N_- = 2\cdot2\times10^7 \text{ from (26)}.$$

Since $(k/\omega)^2 = 0\cdot068$, this justifies the assumption that equation (21), i.e. self-repulsion of the electrons, marks the final stage. Then $i = 2\cdot75\times10^{-4}$ A, from (18). And the duration of the stage is, from (1) 820 half-cycles, or 25 μs at $\lambda = 18$ m.

(ii) *Growth of current in hydrogen at pressure* 2μ

Initial conditions:

$$a = 2{\cdot}6\times10^{-2}, \qquad \delta = \frac{1+a}{1-a}+1{\cdot}5\,\% = 1{\cdot}067, \qquad \delta(1-a) = 1{\cdot}041,$$

$$N_- = 0, \qquad n_+ = 0, \qquad N_+ = 0.$$

Final conditions:

$$N_-/N_+(1-\cos k\pi/\omega) = 1,$$

$$N_-/N_+ = \frac{4{\cdot}1\times10^{-2}}{2{\cdot}6\times10^{-2}} = 1{\cdot}58, \quad \text{from } (2a). \tag{27}$$

Hence $k/\omega = 0{\cdot}38, \quad (k/\omega)^2 = 0{\cdot}144,$

$$n_+ = 4{\cdot}96\times10^5 \text{ from (15)}, \quad N_+ = 1{\cdot}49\times10^7,$$

$$\text{from (27) } N_- = 2{\cdot}36\times10^7 \text{ electrons.}$$

Then $i = 2{\cdot}95\times10^{-4}\,\text{A}$, from (18).

The duration of the growth, from (1) is 420 half-cycles, or 13 μs at $\lambda = 18$ m.

These calculated values of currents as a function of the time are shown in figure 10.

PART II. MEASUREMENTS

8. Introduction

Previous experiments on the high-frequency electrodeless discharge at low pressure (Gill & von Engel 1948) showed the dependence of the starting field upon the wave-length of the applied field. The starting field was found to be independent of the gas and of the pressure within a certain range, and it was shown that the discharge is started by secondary electron emission from the glass walls of the vessel. In part I the development of the discharge from this starting process up to the final equilibrium state has been investigated theoretically, and the rate at which the discharge develops has been deduced.

The purpose of these experiments is to measure the rate of growth of the discharge, and its dependence on the gas, the pressure and the frequency, and hence to check the theory.

9. Principle of experimental method

(a) *Requirements*

The normal method of measuring the rate of growth of a discharge is to apply a potential across the gas and observe how the potential falls when a discharge starts. It has been used extensively in the investigation of d.c. discharges and of discharges at very high frequencies. This method has been rejected in the present work for two reasons. First, the fall of potential is largely a circuit property, and not a fundamental property of the gas discharge itself. Secondly, at these low pressures the fall in potential caused by the gas discharge is very small compared with the starting potential.

It was decided to measure instead the current passing through the gas. This is a more fundamental measurement, as it corresponds to actual movement of charges in the gas. If the values of the circuit components are suitably chosen the rate of growth of this current does not depend upon the circuit.

The main obstacle to the measurement of the gas-discharge current is the large capacitative current that flows between the electrodes which are outside the discharge tube. This capacitative current is much greater than the discharge current. For example, if the external electrodes form a condenser of capacity 30 $\mu\mu$F and a potential of 150 V r.m.s. is applied at a frequency of $\sim$17 Mc/s ($\lambda = 18$ m) the capacitative current is $\sim$0·5 A. The current flowing through the discharge can be estimated by loading the circuit with a resistance or a capacity which induces the same voltage drop as the discharge. It was found that the final (i.e. maximum) discharge current is less than 1 mA. In order to measure the current in the earliest stages of the discharge it is necessary to discriminate one part of gas-discharge current from more than 1000 parts of capacitative current.

A differential method is therefore necessary to balance out the capacitative current flowing across the electrodes.

(*b*) *The circuit*

The circuit used in practice is shown in figure 11. Two similar electrode systems *A* and *B* are connected across a parallel resonant circuit, which is electromagnetically coupled to the high-frequency generator. Electrostatic coupling is avoided by using the perforated earthed screen *S*, thus keeping the electrode systems symmetrical with respect to earth (i.e. their mean potential is zero). The discharge vessel, a hard glass cylinder (Pyrex) of length $d = 6$ cm and diameter 3 cm, is placed between the electrodes *A* with its axis parallel to the field.

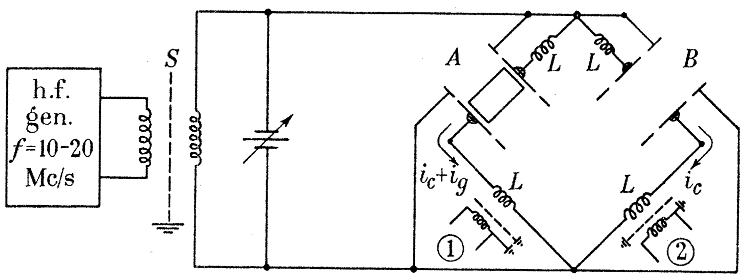


FIGURE 11. Electrode system with guard rings forming a bridge circuit.

In order to reduce the capacitative current the electrodes are made as small as possible —plane parallel disks of diameter 5 cm. To keep the field across the discharge vessel uniform these disks are surrounded by guard rings. (The electrode assembly *B* is made identical.) In series with the inner disks, are connected small coils *L*, across which there is only a very small voltage drop.

Consider now the coils connected to the electrodes *A*. When there is no gas discharge the current through *L* is i_c, essentially determined by the capacity of the inner disks. If a discharge occurs the current through *L* is i_c plus the gas discharge current i_g. More accurately, since the occurrence of the gas discharge will cause a slight change in the voltage across the parallel resonant circuit, i_c will have changed proportionally with the voltage. Thus the total current through *L* will be $\vec{i}'_c + \vec{i}_g$.

Now the coil connected to the electrodes, B, carries a current i_c when there is no gas discharge, and a current i'_c when a gas discharge occurs, the change being due only to the small voltage change across the resonant circuit.

By using two small pick-up coils (1) and (2), electrostatically screened, but coupled magnetically to the coils L, two potentials (1) and (2) are obtained which are strictly proportional to the currents through the coils L, i.e. when no discharge occurs

$$\text{potential (1)} \propto i_c,$$
$$\text{potential (2)} \propto i_c.$$

And when a discharge occurs

$$\text{potential (1)} \propto \vec{i'_c} + \vec{i_g},$$
$$\text{potential (2)} \propto i'_c.$$

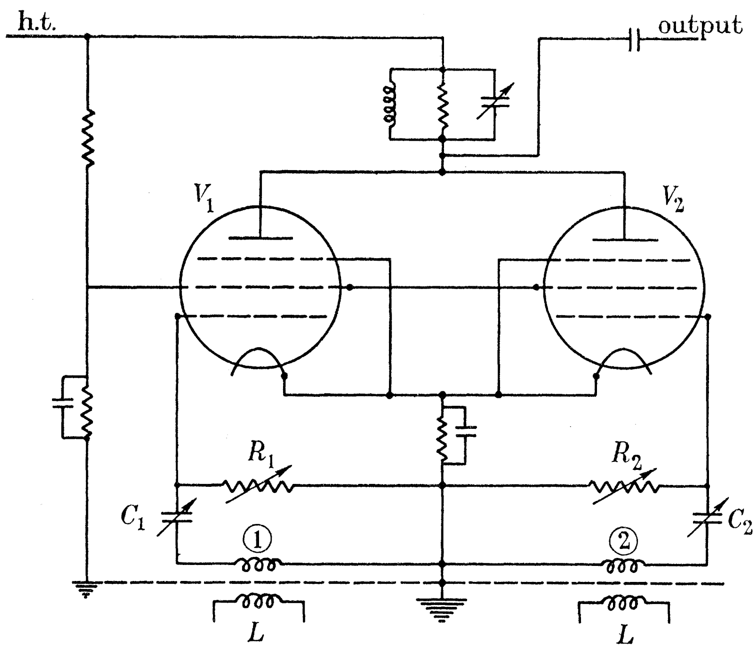


FIGURE 12. Balanced amplifier.

These two potentials can be balanced by applying them in opposition to the balanced amplifier shown in figure 12. Two pentodes are connected in parallel, the potential (1) is applied to the grid of V_1 and potential (2) to the grid of V_2. Since it is not possible to construct the electrode systems A and B, nor the pick-up circuits to be exactly identical, the balancing has to be done at the input of the amplifier by the capacity resistance networks C_1, R_1 and C_2, R_2. Here C_1, C_2 determine the magnitudes of the potentials applied to the grid, and R_1, R_2 can be adjusted to compensate for slight deviations from the 180° phase difference between the applied potentials.

When there is no discharge the networks C_1R_1 and C_2R_2 can be adjusted to make the output from the amplifier zero. If a discharge then occurs the circuit becomes unbalanced and the voltage output from the amplifier is strictly proportional to the gas discharge current i_g. This output, after amplification, can be displayed on an oscilloscope.

All the circuit components were carefully chosen, and their self-capacities and self-inductances considered to avoid any possible resonances at or near the frequencies used.

This circuit enables the discharge current to be measured. In order to measure the rate of growth of this current a pulse technique is necessary, and the circuit shown in figure 13 is used. A pulse generator governs the high-frequency oscillator and causes pulses of high-frequency potential to be applied across the electrodes. These pulses are repeated at regular intervals, determined by an asymmetrical multivibrator which triggers both the pulse generator and the time-base of the cathode-ray oscilloscope. Thus the traces of these voltage pulses are superimposed on the oscilloscope screen and form a stationary picture. The output from the balanced amplifier is also displayed on the screen after suitable amplification and rectification. Thus both the potential across the discharge tube and the current passing through the discharge can be observed simultaneously.

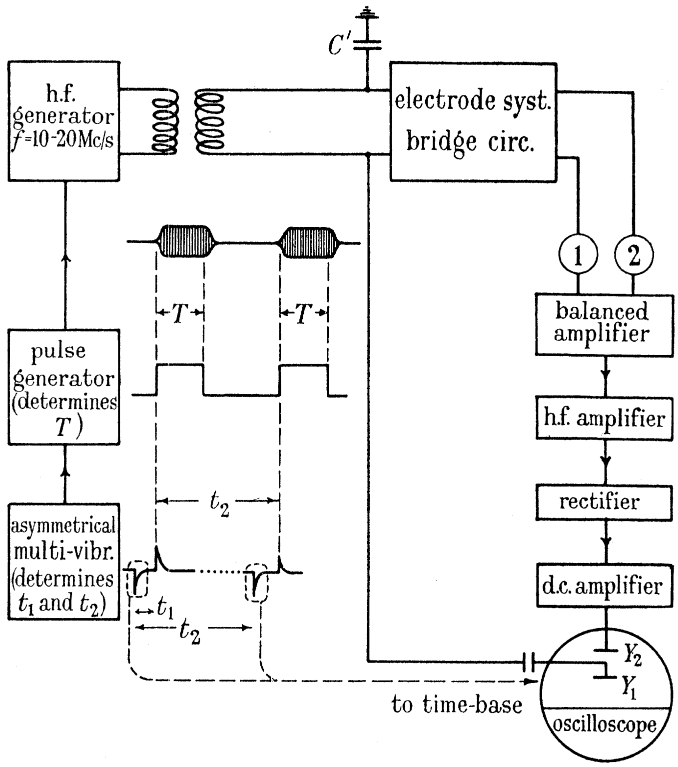


FIGURE 13. Scheme of the circuit for displaying the growth of the current. The trace obtained is one-half of the envelope of the radio-frequency current in the discharge. Delay time t_1 variable, 10 to 50 μs; pulse interval t_2 variable, 100 to 50 ms; pulse length T variable, 0 to 100 μs.

The condenser C' in figure 13 balances the capacity of the Y_1 plates of the oscilloscope and maintains the symmetry of the circuit with respect to earth. Resistances connected across the coils L (not shown in figure 12) damp circuit transients should the discharge current rise very quickly. All time constants in the amplifier circuits were designed to be very much less than the fastest possible rate of increase in the discharge current. All the amplifiers and the coils (1) and (2) are most carefully screened to avoid unwanted pick-up from the electrode system or the h.f. generator.

10. EXPERIMENTAL PROCEDURE

A steady potential is induced into the parallel resonant circuit from the h.f. generator, and the circuit tuned to resonance. The h.f. generator is then pulsed at any convenient repetition rate, and any necessary slight adjustments made to ensure that the pulse is of constant height. The potential is adjusted until it is just less than the starting potential of the discharge.

The current-measuring circuit will not in general be balanced. By adjusting C_1, C_2 R_1, R_2, zero output can be obtained from the amplifiers; the circuit is then properly balanced. A test is made to see that this balancing condition does not depend on the voltage across the electrodes. The voltage is varied (though not allowed to exceed starting potential), and it is found that the output from the amplifiers remains zero.

The potential is now increased. When it becomes equal to starting potential, discharges start when the pulses are applied. There is then an output from the balanced amplifier, and traces appear on the screen as shown later in the oscillograms. The shape of these traces represents the growth of the gas-discharge current with time. A discharge will not start at the same instant during every pulse, but at times which are statistically distributed. Thus one observes on the screen a large number of exactly similar traces starting at different times after the voltage has been applied.

11. Calibration of the circuit

The ideal method of calibrating the circuit would be to replace the gas discharge by a known impedance (resistance or condenser), and observe the displacement of the trace on the screen. The voltage across the electrodes could then be measured by means of two diodes charging an electrostatic voltmeter (Gill & von Engel, 1948). The current drawn by the impedance could thus be calculated. This method has not proved possible in practice. A minute capacity ($\simeq \frac{1}{50}$ pF) is required to give a reasonable deflexion, and a capacity of this size is hard to make, and harder to measure. Resistances are not suitable because their capacities vary considerably in different resistances.

An indirect method therefore had to be used. The inductance of the coil L and its connexions was first measured, and its impedance calculated at the particular frequency. The h.f. generator was then switched off. A potential from a standard signal generator of the same frequency as that applied to the gas discharge is applied across the coil L. (As a further check the potential across L was measured by a valve voltmeter.) The current flowing through the coil could be calculated, and observed to correspond to a particular displacement of the trace on the screen.

12. Measurement of intensity of light

A measurement of the distribution of light across the discharge vessel yields information about the speed of the electrons in the final stages of the growth of the discharge when most of the light is produced. One can observe by eye the appearance of the discharge, but more precise information is needed to deduce the speed of the electrons.

If we assume that in the final state the energy of the electrons never exceeds about 50 eV (figure 8), then in those regions where the electrons are fastest the ionization produced is greatest and the number of atoms excited to high energies will be largest. This is because at energies >50 eV both excitation to higher levels and ionization increases similarly with the electron energy (Bates, Fundaminsky, Leech & Massey 1950). Hence most of the violet and ultra-violet light is emitted from these regions of the discharge tube. The distribution of blue light across the tube, which therefore corresponds approximately to the distribution of ions, was measured qualitatively as shown in figure 14*a*.

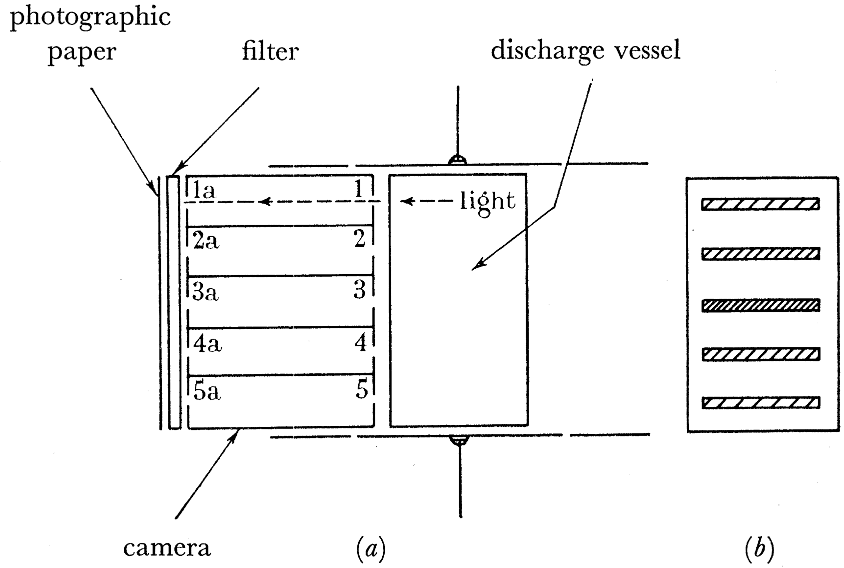


FIGURE 14*a*. Rough measurement of the distribution of light emitted by the discharge.

FIGURE 14*b*. Distribution of light in the fully developed discharge for helium and hydrogen.

A cardboard camera consisting of two series of slits (1, 2, 3, 4, 5; 1*a*, 2*a*, 3*a*, 4*a*, 5*a*) in line with each other was placed close to the discharge tube. The two sets of slits are joined by divisions which ensures that light can travel only through corresponding slits, e.g. 1, 1*a*; 2, 2*a*, etc., and cannot travel crosswise. A filter which transmits only blue light and a piece of photographic paper are placed behind the slits. A continuous discharge was switched on and the photographic paper exposed. Images of the slits appeared on the paper, their densities corresponding to the light intensities at various parts of the discharge tube (figure 14*b*). The camera, being an insulator and made of thin material, hardly affects the field across the electrodes.

13. Observations, and discussion of results

The mechanism by which the discharge develops from the initial process (i.e. secondary electron emission from the walls of the vessel) up to the final state has been investigated theoretically in the first part of this paper. The rate of growth of the current through the discharge has been calculated and the theory shows that as various parameters are altered the discharge should behave in the manner described below.

(*a*) *Growth of current in general*

From the theory, one would expect the current to grow in three distinct stages. During the first stage the current should rise according to the relation $i \propto \delta^{\alpha t}$, where δ is a constant just greater than 1. This should be followed by a period when the current rises more slowly than a function of the form $\delta^{\alpha t}$, and to a first approximation linearly with time except at very low pressure. Finally the current should become constant, corresponding to the final equilibrium state of the discharge.

These three stages can be recognized in the oscillograms shown in figure 15, which were taken from helium and hydrogen discharges respectively, each at a pressure of 12×10^{-3} mm Hg; starting potential was applied at a wave-length of 18 m. The growth of the discharge in helium and hydrogen for pressures 2 and 12μ has been calculated in detail, and figure 10

helium hydrogen

$p =$ 12μ

2μ

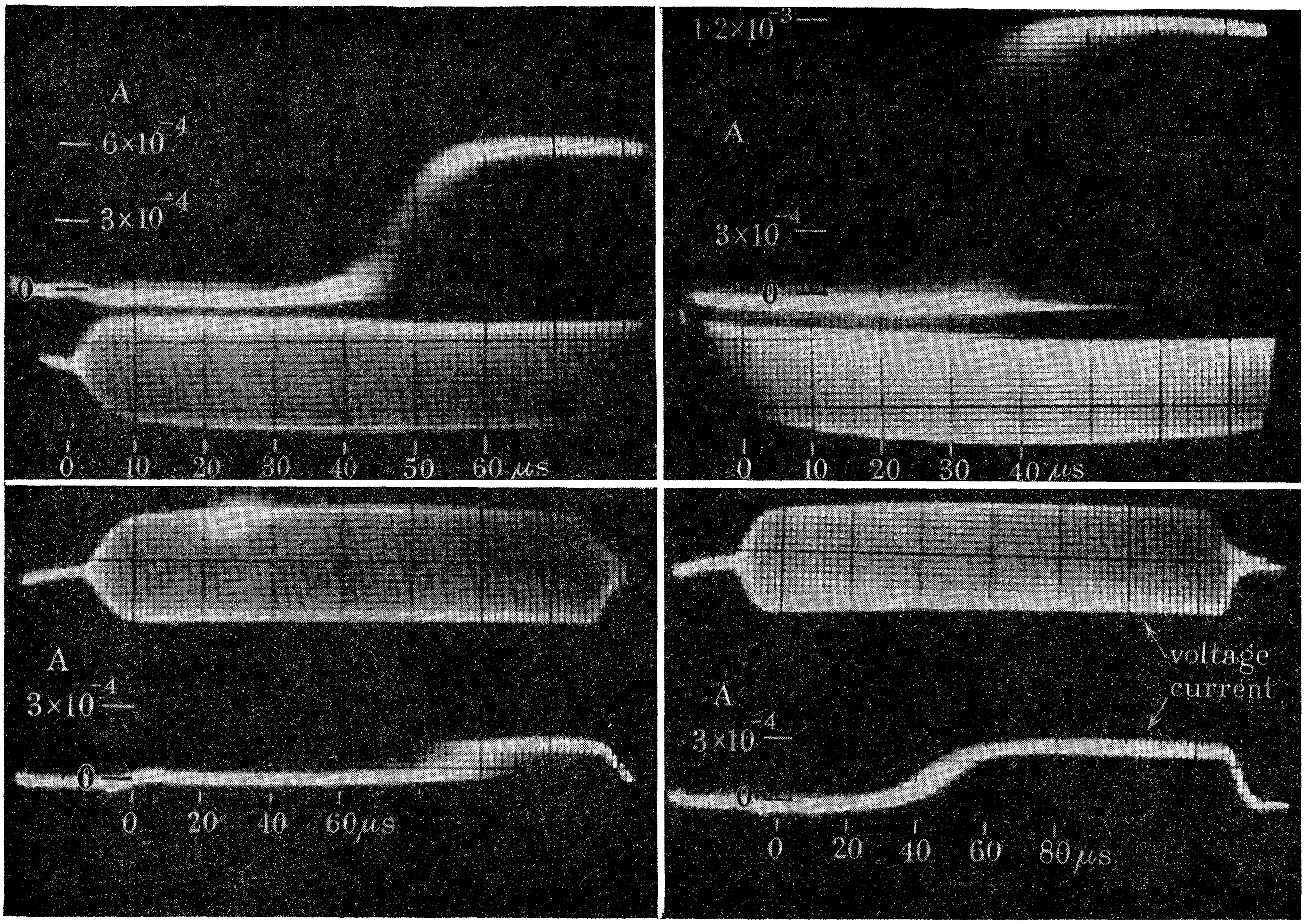


FIGURE 15. Oscillograms of the current as a function of the time for hydrogen and helium at 2 and 12μ, at starting potential. (The trace of the voltage pulse is distorted because of the curvature of the oscilloscope screen).

shows both the oscillograms and the theoretical curves. It can be seen that the agreement is satisfactory but precise results cannot be expected from oscillograms of this kind. However, measurements taken from enlarged oscillograms show that in the later stages the rise of current actually found is always slightly slower than that expected theoretically. This is probably due to the over-simplifications in the theory, particularly the neglect of certain appreciable losses of electrons, e.g. due to inelastic scattering which deflects them to the side walls.

It will be observed that the voltage drop caused by the starting of the discharge is hardly visible in any of the oscillograms.

(*b*) *Distribution of ions and excited atoms in the discharge*

In the final stage of the discharge it seems that the electrons oscillate with an amplitude less than the length of the vessel, being fastest at the centre where they have an energy slightly greater than the ionization energy of the gas. Ions, and excited atoms should therefore be most concentrated near the centre; there should be hardly any near the walls. This is confirmed by figure 14*b*, which records the distribution of blue and violet light emitted from the fully developed discharge. The distribution of light was found to be independent of the discharge current within a fairly wide range of current.

(*c*) *Effect of the pressure and the nature of the gas*

A change in the rate of production of ions and electrons in the gas would according to the theory, cause a marked change in the rate of growth of current. This can be produced either by changing the pressure of the gas already in the vessel, or by replacing it by another gas which has a markedly different probability of ionization by electrons. Increasing the pressure of the gas, or replacing, say, helium by hydrogen (both of which increase the production of ions) should cause the current to rise faster, as well as to a larger constant value. Also the second stage of the growth, which is largely affected by the ion space charge, would be expected to appear sooner. These effects can be seen in the oscillograms, figure 15. Observations were extended up to 35μ showing the same trends.

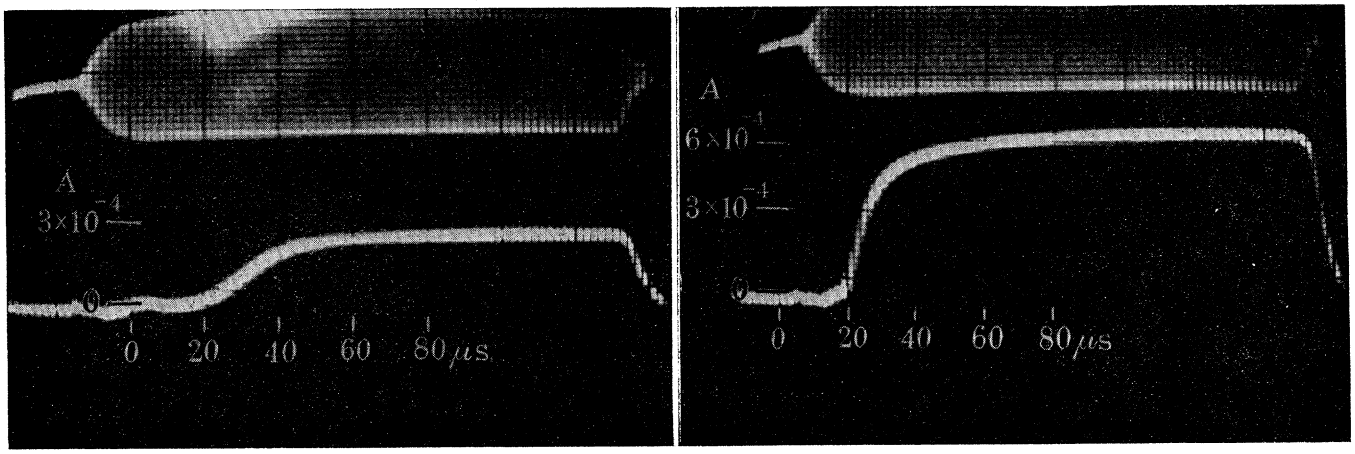
(*d*) *Effect of an excess voltage*

The oscillograms already mentioned have all been taken with starting potential applied to the discharge. If the applied potential is increased, the speed with which electrons hit the end walls in the early stages of the discharge will also be increased, and hence the secondary emission. It has been shown that secondary emission is the dominant process in the first stage of the discharge, when the current rises proportionally to $\delta^{\alpha t}$, and it remains an effective although diminishing process throughout the second stage of the growth. Since δ is only just greater than 1 a small increase in δ should cause a rapid increase in $\delta^{\alpha t}$, and hence in the rate of growth of current. A small excess voltage should therefore cause a much faster rise of current in the first two stages. The final current will also be increased, because the electrons will be slightly faster in the final state thus producing more ionization.

The experimentally observed effect of a small excess voltage (5 to 20 % above starting potential) is shown in the oscillograms, figure 16.

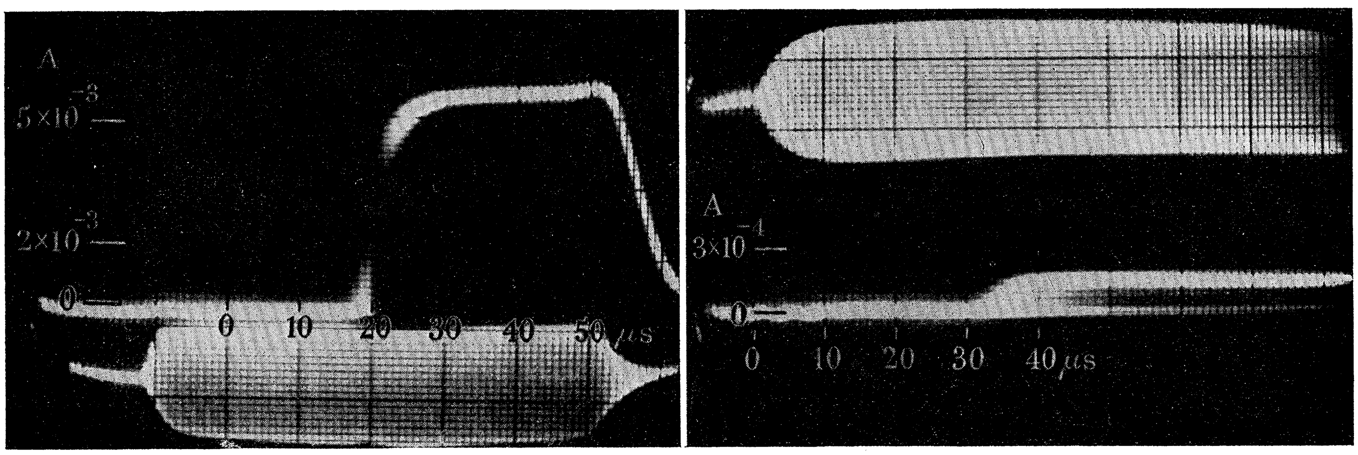
(*e*) *Effect of the frequency*

In the first two stages the motion of the electrons and hence the processes of production (secondary emission and gas ionization) have been shown to depend on the parameters X and ϕ, where X is the starting field and ϕ the phase in which an electron leaves an end wall. A change of frequency alters both X and ϕ (Gill & von Engel 1948): X increases and ϕ becomes more positive when the frequency increases. It can be shown from the equations of motion, plotting the energy of the electrons as a function of their distance from one wall of the vessel (a figure analogous to figure 5), that raising the frequency considerably increases the ionization in the gas. The minimum δ for starting is thus larger (see (2*b*)) and the rate of growth which is initially $\alpha\delta^{\alpha t}$ increases rapidly. At a later stage when many ions are present losses must be considered. The losses during any half-cycle of the applied field, notably by self-repulsion, are proportional to the duration of a half cycle. They are large at low frequencies and vice versa. Since the resultant growth of current is the difference between production and losses, it should be faster at high frequencies than at low frequencies.



5 % 20 % excess voltage

FIGURE 16. Change of the rate of growth of current with the excess voltage (5 and 20 %) in helium at 2μ. Compare also the rate of growth at starting potential as shown in figure 15 and the different scatter in the rising part.



$\lambda = 15$ m $\lambda = 23$ m

FIGURE 17. Change of the rate of growth of current with the frequency in hydrogen at 12μ for wave-lengths of 15 and 23 m at starting potential.

In the final stage it appears that the maximum velocity of the electrons as they cross the middle of the vessel will be roughly proportional to the frequency, and hence the energy is proportional to the square of the frequency. This maximum energy is only slightly greater than the ionization energy of the gas. A small increase in frequency can therefore cause a large increase in this excess energy, hence in the production of new ions and electrons, and thus in the final current. The equilibrium current in the final stage should then be most sensitive to changes in the frequency; a change of frequency by a factor 1·5 might easily cause the final current to change by an order of magnitude or more.

The experimental results confirming qualitatively these conclusions are shown in the oscillograms, figure 17.

(*f*) *Effect of statistical fluctuations*

It has already been explained that the starting of the discharge depends on a chance electron being at the right place, preferably near an end-wall, in a favourable phase of the field. The discharge is not likely to start at the same instant during the application of each pulse. The oscillograms shown are the traces of a large number of pulses, and the blurred image of the rise of current in many of them shows that the discharges started at random times after the application of the pulse.

It should be possible to reduce these statistical fluctuations, either by illuminating an end wall of the vessel with ultra-violet light, where photo-electrons will be produced, or by reducing the time interval between pulses, so that ions and electrons left from a previous discharge may start the next one. Provided that the number of ions and electrons then remaining in the vessel is not too great (i.e. provided that the interval between pulses is greater than the time ions take to drift to the walls and recombine, viz. 100 μs) there should be no effect upon the rate of growth of the discharge, or upon the starting potential.

This was found to be so experimentally. The interval between pulses was changed from 100 μs to 50 ms. No change was observed in the shape of the trace, but statistical fluctuations in the starting point of the trace increased enormously. Illumination with ultra-violet light reduced statistical fluctuations, but had no other effect.

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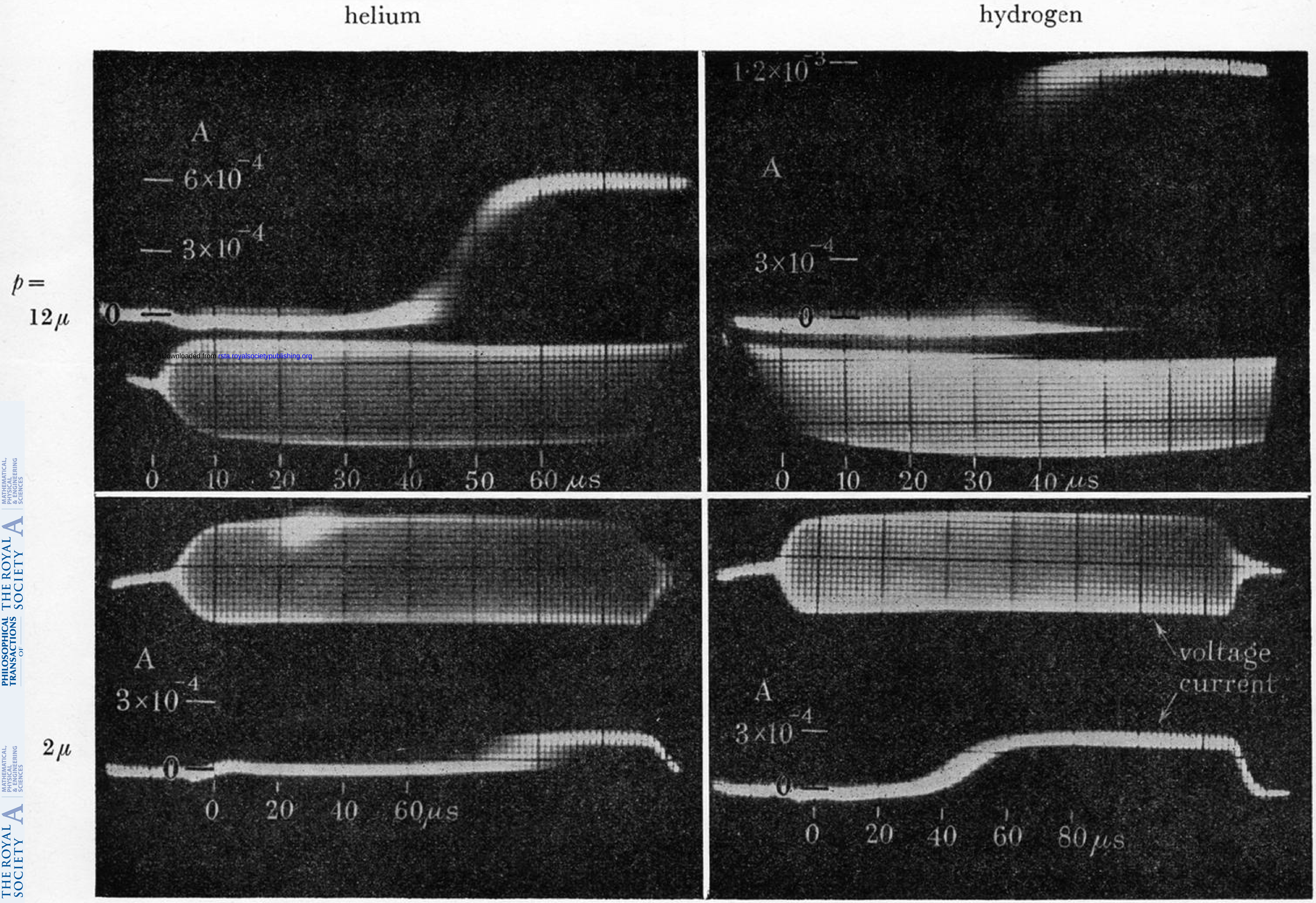


FIGURE 15. Oscillograms of the current as a function of the time for hydrogen and helium at 2 and $12\,\mu$, at starting potential. (The trace of the voltage pulse is distorted because of the curvature of the oscilloscope screen).

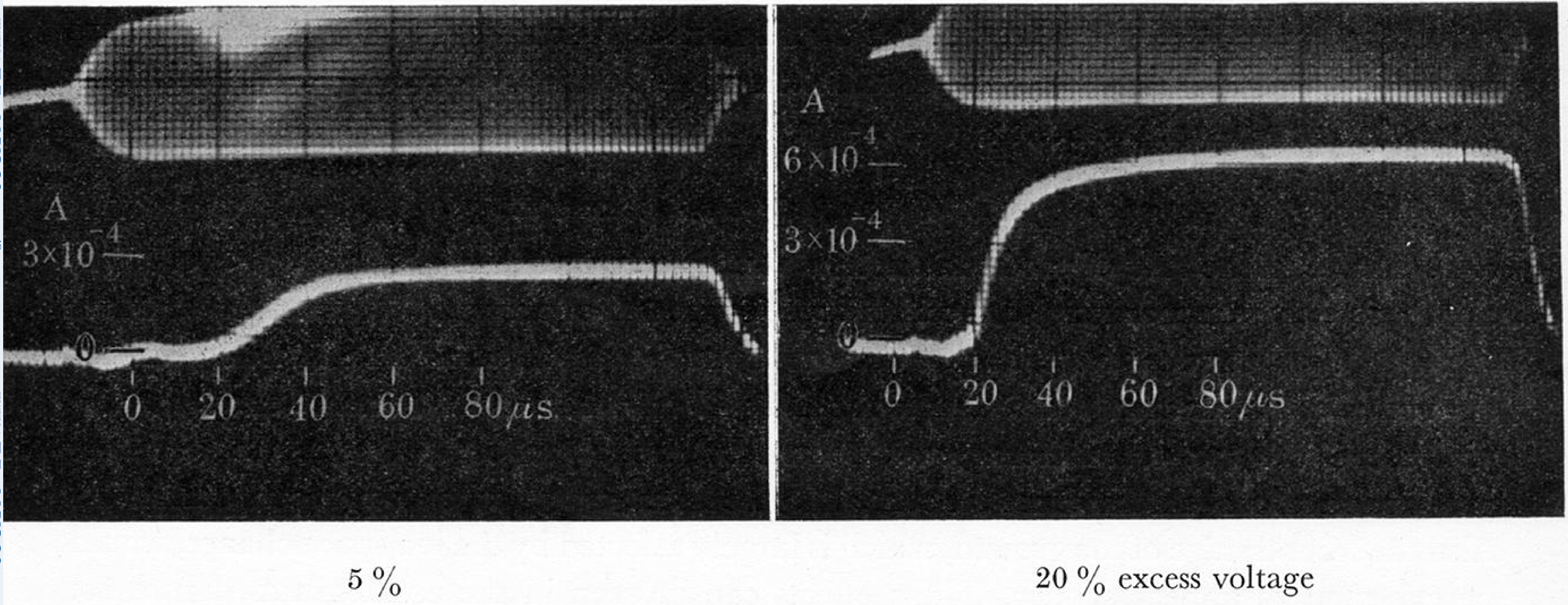
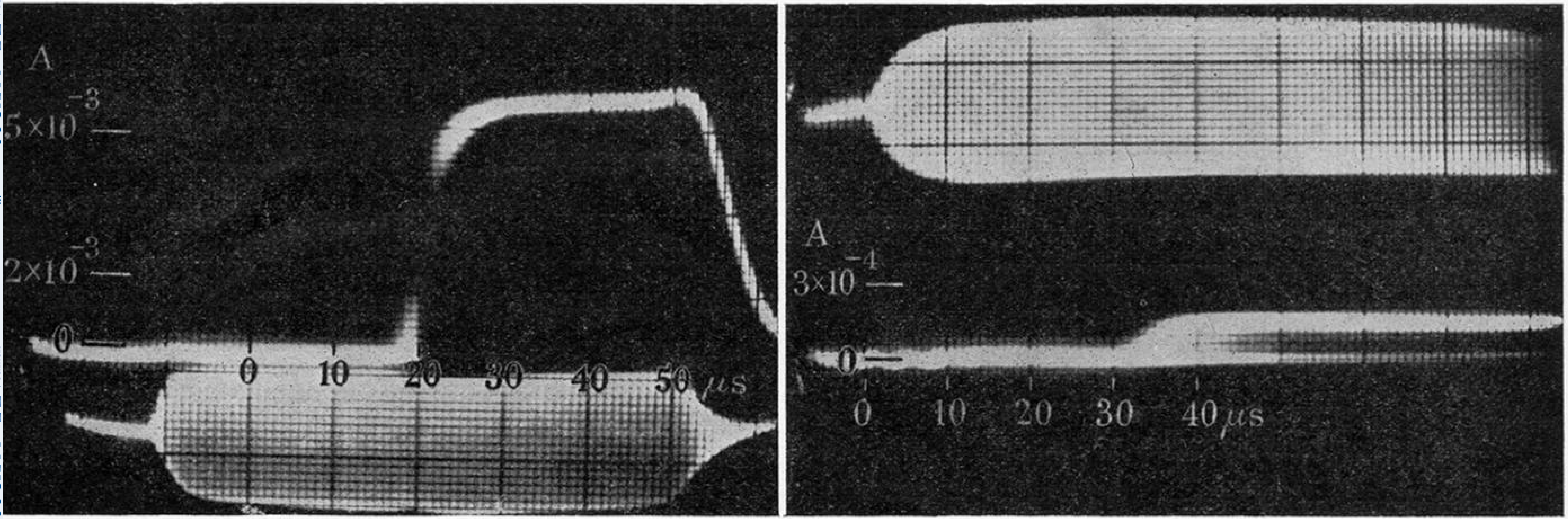


FIGURE 16. Change of the rate of growth of current with the excess voltage (5 and 20 %) in helium at 2 μ. Compare also the rate of growth at starting potential as shown in figure 15 and the different scatter in the rising part.



λ = 15 m λ = 23 m

FIGURE 17. Change of the rate of growth of current with the frequency in hydrogen at 12 μ for wave-lengths of 15 and 23 m at starting potential.